2 | GEOMETRIC OPTICS AND IMAGE FORMATION

Figure 2.1 *Cloud Gate* is a public sculpture by Anish Kapoor located in Millennium Park in Chicago. Its stainless steel plates reflect and distort images around it, including the Chicago skyline. Dedicated in 2006, it has become a popular tourist attraction, illustrating how art can use the principles of physical optics to startle and entertain. (credit: modification of work by Dhilung Kirat)

Chapter Outline

Introduction

This chapter introduces the major ideas of geometric optics, which describe the formation of images due to reflection and refraction. It is called “geometric” optics because the images can be characterized using geometric constructions, such as ray diagrams. We have seen that visible light is an electromagnetic wave; however, its wave nature becomes evident only when light interacts with objects with dimensions comparable to the wavelength (about 500 nm for visible light). Therefore, the laws of geometric optics only apply to light interacting with objects much larger than the wavelength of the light.

2.1 | Images Formed by Plane Mirrors

Learning Objectives

By the end of this section, you will be able to:

- Describe how an image is formed by a plane mirror.
- Distinguish between real and virtual images.
- Find the location and characterize the orientation of an image created by a plane mirror.

You only have to look as far as the nearest bathroom to find an example of an image formed by a mirror. Images in a **plane mirror** are the same size as the object, are located behind the mirror, and are oriented in the same direction as the object (i.e., "upright").

To understand how this happens, consider Figure 2.2. Two rays emerge from point P, strike the mirror, and reflect into the observer's eye. Note that we use the law of reflection to construct the reflected rays. If the reflected rays are extended backward behind the mirror (see dashed lines in Figure 2.2), they seem to originate from point Q. This is where the image of point P is located. If we repeat this process for point P', we obtain its image at point Q'. You should convince yourself by using basic geometry that the image height (the distance from Q to Q') is the same as the object height (the distance from P to P'). By forming images of all points of the object, we obtain an upright image of the object behind the mirror.

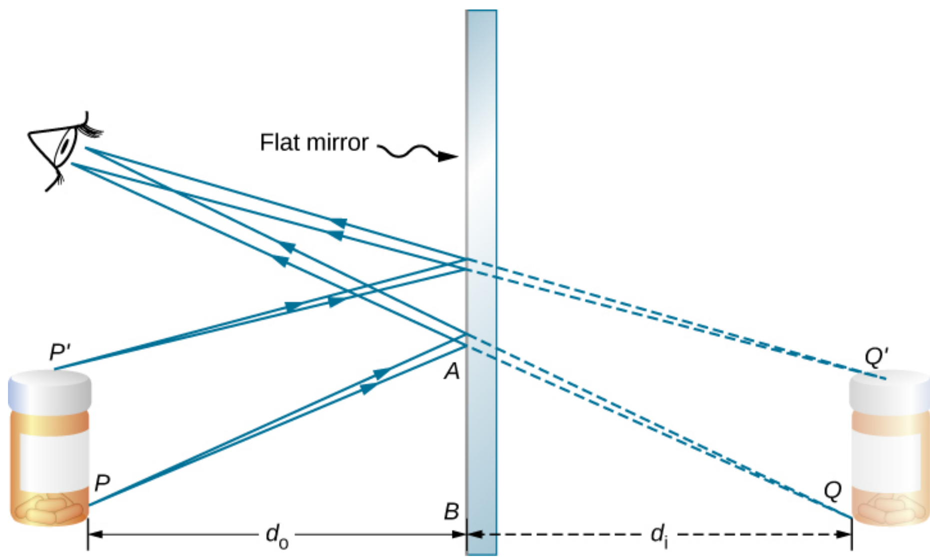


Figure 2.2 Two light rays originating from point P on an object are reflected by a flat mirror into the eye of an observer. The reflected rays are obtained by using the law of reflection. Extending these reflected rays backward, they seem to come from point Q behind the mirror, which is where the virtual image is located. Repeating this process for point P' gives the image point Q'. The image height is thus the same as the object height, the image is upright, and the object distance d_o is the same as the image distance d_i. (credit: modification of work by Kevin Dufendach)

Notice that the reflected rays appear to the observer to come directly from the image behind the mirror. In reality, these rays come from the points on the mirror where they are reflected. The image behind the mirror is called a **virtual image** because it cannot be projected onto a screen—the rays only appear to originate from a common point behind the mirror. If you walk behind the mirror, you cannot see the image, because the rays do not go there. However, in front of the mirror, the rays behave exactly as if they come from behind the mirror, so that is where the virtual image is located.

Later in this chapter, we discuss real images; a **real image** can be projected onto a screen because the rays physically go through the image. You can certainly see both real and virtual images. The difference is that a virtual image cannot be projected onto a screen, whereas a real image can.

Locating an Image in a Plane Mirror

The law of reflection tells us that the angle of incidence is the same as the angle of reflection. Applying this to triangles *PAB* and *QAB* in **Figure 2.2** and using basic geometry shows that they are congruent triangles. This means that the distance *PB* from the object to the mirror is the same as the distance *BQ* from the mirror to the image. The **object distance** (denoted d_o) is the distance from the mirror to the object (or, more generally, from the center of the optical element that creates its image). Similarly, the **image distance** (denoted d_i) is the distance from the mirror to the image (or, more generally, from the center of the optical element that creates it). If we measure distances from the mirror, then the object and image are in opposite directions, so for a plane mirror, the object and image distances should have the opposite signs:

$$d_\text{o} = -d_\text{i}. \tag{2.1}$$

An extended object such as the container in **Figure 2.2** can be treated as a collection of points, and we can apply the method above to locate the image of each point on the extended object, thus forming the extended image.

Multiple Images

If an object is situated in front of two mirrors, you may see images in both mirrors. In addition, the image in the first mirror may act as an object for the second mirror, so the second mirror may form an image of the image. If the mirrors are placed parallel to each other and the object is placed at a point other than the midpoint between them, then this process of image-of-an-image continues without end, as you may have noticed when standing in a hallway with mirrors on each side. This is shown in **Figure 2.3**, which shows three images produced by the blue object. Notice that each reflection reverses front and back, just like pulling a right-hand glove inside out produces a left-hand glove (this is why a reflection of your right hand is a left hand). Thus, the fronts and backs of images 1 and 2 are both inverted with respect to the object, and the front and back of image 3 is inverted with respect to image 2, which is the object for image 3.

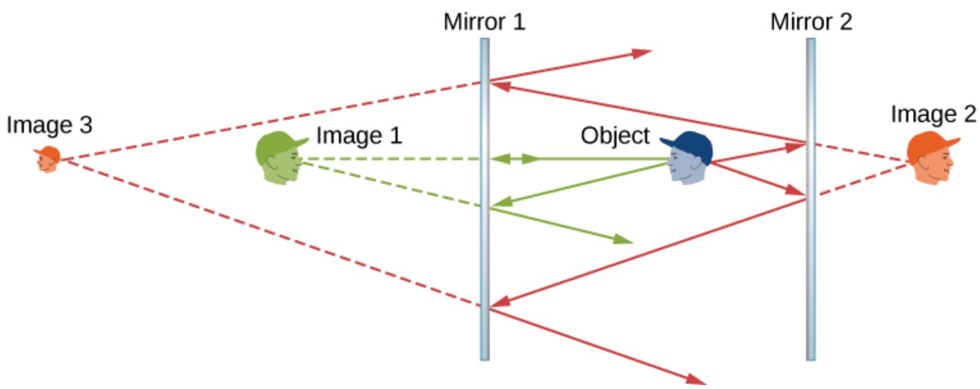


Figure 2.3 Two parallel mirrors can produce, in theory, an infinite number of images of an object placed off center between the mirrors. Three of these images are shown here. The front and back of each image is inverted with respect to its object. Note that the colors are only to identify the images. For normal mirrors, the color of an image is essentially the same as that of its object.

You may have noticed that image 3 is smaller than the object, whereas images 1 and 2 are the same size as the object. The ratio of the image height with respect to the object height is called **magnification**. More will be said about magnification in the next section.

Infinite reflections may terminate. For instance, two mirrors at right angles form three images, as shown in part (a) of **Figure 2.4**. Images 1 and 2 result from rays that reflect from only a single mirror, but image 1,2 is formed by rays that reflect from both mirrors. This is shown in the ray-tracing diagram in part (b) of **Figure 2.4**. To find image 1,2, you have to look behind the corner of the two mirrors.

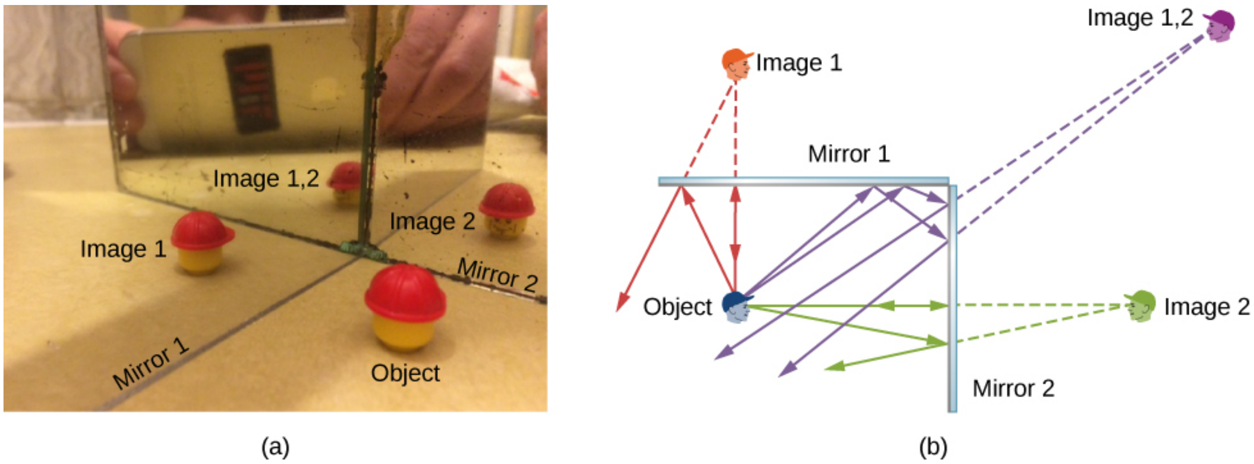


Figure 2.4 Two mirrors can produce multiple images. (a) Three images of a plastic head are visible in the two mirrors at a right angle. (b) A single object reflecting from two mirrors at a right angle can produce three images, as shown by the green, purple, and red images.

2.2 | Spherical Mirrors

Learning Objectives

By the end of this section, you will be able to:

- Describe image formation by spherical mirrors.
- Use ray diagrams and the mirror equation to calculate the properties of an image in a spherical mirror.

The image in a plane mirror has the same size as the object, is upright, and is the same distance behind the mirror as the object is in front of the mirror. A **curved mirror**, on the other hand, can form images that may be larger or smaller than the object and may form either in front of the mirror or behind it. In general, any curved surface will form an image, although some images make be so distorted as to be unrecognizable (think of fun house mirrors).

Because curved mirrors can create such a rich variety of images, they are used in many optical devices that find many uses. We will concentrate on spherical mirrors for the most part, because they are easier to manufacture than mirrors such as parabolic mirrors and so are more common.

Curved Mirrors

We can define two general types of spherical mirrors. If the reflecting surface is the outer side of the sphere, the mirror is called a **convex mirror**. If the inside surface is the reflecting surface, it is called a **concave mirror**.

Symmetry is one of the major hallmarks of many optical devices, including mirrors and lenses. The symmetry axis of such optical elements is often called the principal axis or **optical axis**. For a spherical mirror, the optical axis passes through the mirror's center of curvature and the mirror's **vertex**, as shown in **Figure 2.5**.

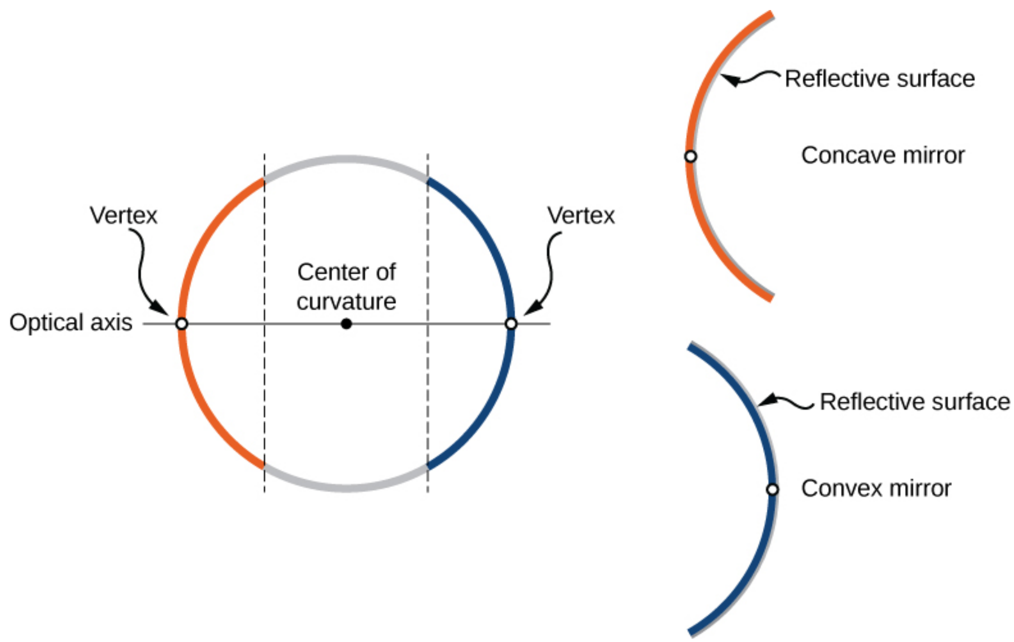


Figure 2.5 A spherical mirror is formed by cutting out a piece of a sphere and silvering either the inside or outside surface. A concave mirror has silvering on the interior surface (think “cave”), and a convex mirror has silvering on the exterior surface.

Consider rays that are parallel to the optical axis of a parabolic mirror, as shown in part (a) of **Figure 2.6**. Following the law of reflection, these rays are reflected so that they converge at a point, called the **focal point**. Part (b) of this figure shows a spherical mirror that is large compared with its radius of curvature. For this mirror, the reflected rays do not cross at the same point, so the mirror does not have a well-defined focal point. This is called spherical aberration and results in a blurred image of an extended object. Part (c) shows a spherical mirror that is small compared to its radius of curvature. This mirror is a good approximation of a parabolic mirror, so rays that arrive parallel to the optical axis are reflected to a well-defined focal point. The distance along the optical axis from the mirror to the focal point is called the **focal length** of the mirror.

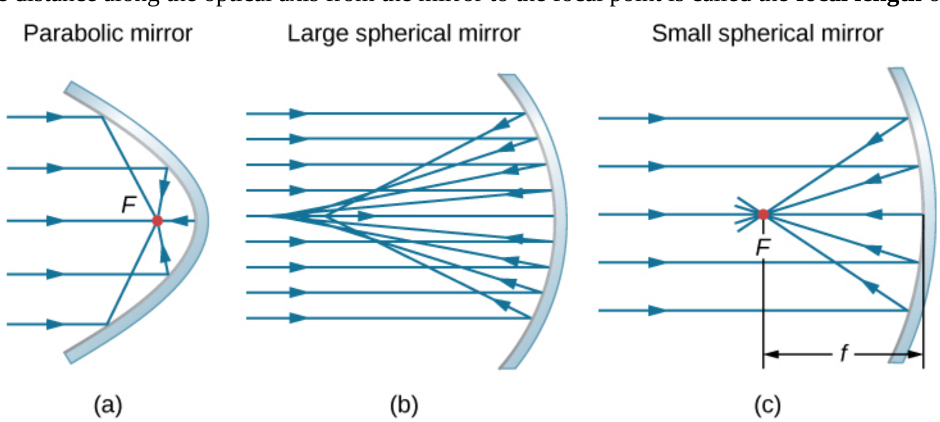


Figure 2.6 (a) Parallel rays reflected from a parabolic mirror cross at a single point called the focal point F. (b) Parallel rays reflected from a large spherical mirror do not cross at a common point. (c) If a spherical mirror is small compared with its radius of curvature, it better approximates the central part of a parabolic mirror, so parallel rays essentially cross at a common point. The distance along the optical axis from the mirror to the focal point is the focal length f of the mirror.

A convex spherical mirror also has a focal point, as shown in **Figure 2.7**. Incident rays parallel to the optical axis are reflected from the mirror and seem to originate from point F at focal length f behind the mirror. Thus, the focal point is virtual because no real rays actually pass through it; they only appear to originate from it.

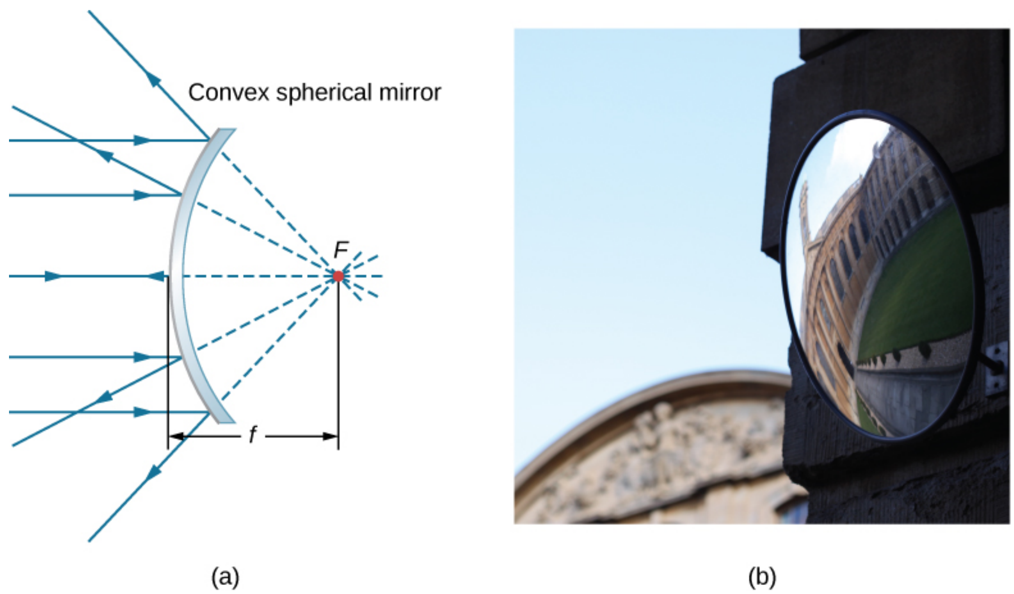


Figure 2.7 (a) Rays reflected by a convex spherical mirror: Incident rays of light parallel to the optical axis are reflected from a convex spherical mirror and seem to originate from a well-defined focal point at focal distance *f* on the opposite side of the mirror. The focal point is virtual because no real rays pass through it. (b) Photograph of a virtual image formed by a convex mirror. (credit b: modification of work by Jenny Downing)

How does the focal length of a mirror relate to the mirror's radius of curvature? **Figure 2.8** shows a single ray that is reflected by a spherical concave mirror. The incident ray is parallel to the optical axis. The point at which the reflected ray crosses the optical axis is the focal point. Note that all incident rays that are parallel to the optical axis are reflected through the focal point—we only show one ray for simplicity. We want to find how the focal length *FP* (denoted by *f*) relates to the radius of curvature of the mirror, *R*, whose length is $R = CF + FP$. The law of reflection tells us that angles *OXC* and *CXF* are the same, and because the incident ray is parallel to the optical axis, angles *OXC* and *XCP* are also the same. Thus, triangle *CXF* is an isosceles triangle with $CF = FX$. If the angle θ is small (so that $\sin\theta \approx \theta$; this is called the "small-angle approximation"), then $FX \approx FP$ or $CF \approx FP$. Inserting this into the equation for the radius *R*, we get

$$R = CF + FP = FP + FP = 2FP = 2f$$

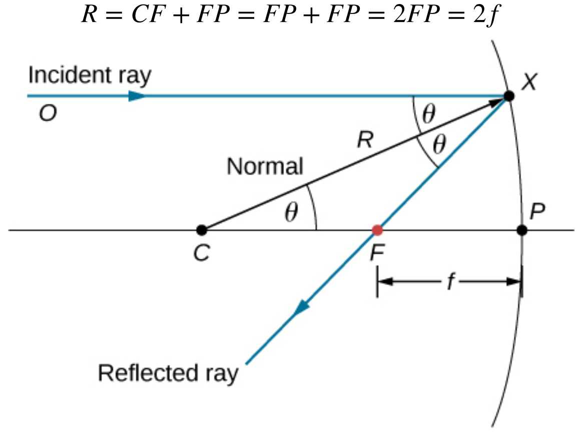


Figure 2.8 Reflection in a concave mirror. In the small-angle approximation, a ray that is parallel to the optical axis *CP* is reflected through the focal point *F* of the mirror.

In other words, in the small-angle approximation, the focal length *f* of a concave spherical mirror is half of its radius of curvature, *R*:

$$f = \frac{R}{2}. \tag{2.2}$$

In this chapter, we assume that the **small-angle approximation** (also called the paraxial approximation) is always valid. In this approximation, all rays are paraxial rays, which means that they make a small angle with the optical axis and are at a distance much less than the radius of curvature from the optical axis. In this case, their angles θ of reflection are small angles, so $\sin\theta \approx \tan\theta \approx \theta$.

Using Ray Tracing to Locate Images

To find the location of an image formed by a spherical mirror, we first use ray tracing, which is the technique of drawing rays and using the law of reflection to determine the reflected rays (later, for lenses, we use the law of refraction to determine refracted rays). Combined with some basic geometry, we can use ray tracing to find the focal point, the image location, and other information about how a mirror manipulates light. In fact, we already used ray tracing above to locate the focal point of spherical mirrors, or the image distance of flat mirrors. To locate the image of an object, you must locate at least two points of the image. Locating each point requires drawing at least two rays from a point on the object and constructing their reflected rays. The point at which the reflected rays intersect, either in real space or in virtual space, is where the corresponding point of the image is located. To make ray tracing easier, we concentrate on four "principal" rays whose reflections are easy to construct.

Figure 2.9 shows a concave mirror and a convex mirror, each with an arrow-shaped object in front of it. These are the objects whose images we want to locate by ray tracing. To do so, we draw rays from point *Q* that is on the object but not on the optical axis. We choose to draw our ray from the tip of the object. Principal ray 1 goes from point *Q* and travels parallel to the optical axis. The reflection of this ray must pass through the focal point, as discussed above. Thus, for the concave mirror, the reflection of principal ray 1 goes through focal point *F*, as shown in part (b) of the figure. For the convex mirror, the backward extension of the reflection of principal ray 1 goes through the focal point (i.e., a virtual focus). Principal ray 2 travels first on the line going through the focal point and then is reflected back along a line parallel to the optical axis. Principal ray 3 travels toward the center of curvature of the mirror, so it strikes the mirror at normal incidence and is reflected back along the line from which it came. Finally, principal ray 4 strikes the vertex of the mirror and is reflected symmetrically about the optical axis.

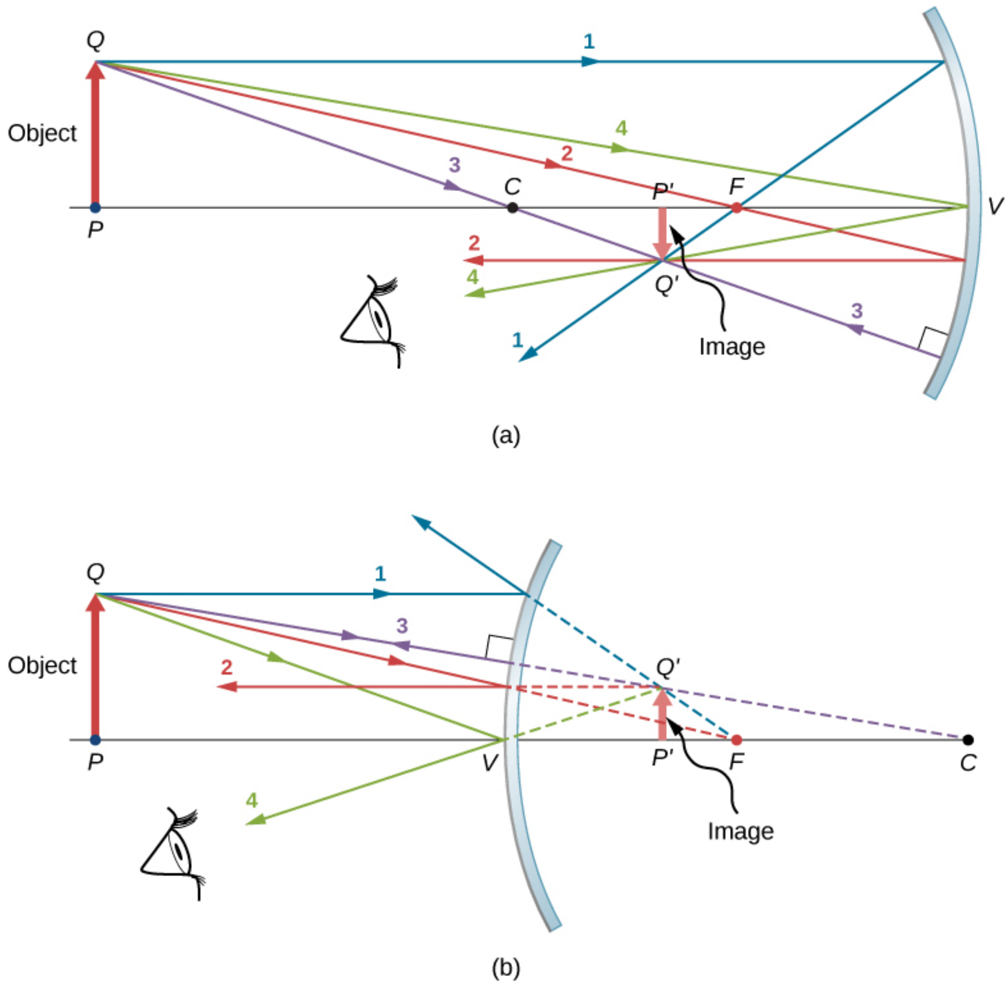


Figure 2.9 The four principal rays shown for both (a) a concave mirror and (b) a convex mirror. The image forms where the rays intersect (for real images) or where their backward extensions intersect (for virtual images).

The four principal rays intersect at point Q', which is where the image of point *Q* is located. To locate point Q', drawing any two of these principle rays would suffice. We are thus free to choose whichever of the principal rays we desire to locate the image. Drawing more than two principal rays is sometimes useful to verify that the ray tracing is correct.

To completely locate the extended image, we need to locate a second point in the image, so that we know how the image is oriented. To do this, we trace the principal rays from the base of the object. In this case, all four principal rays run along the optical axis, reflect from the mirror, and then run back along the optical axis. The difficulty is that, because these rays are collinear, we cannot determine a unique point where they intersect. All we know is that the base of the image is on the optical axis. However, because the mirror is symmetrical from top to bottom, it does not change the vertical orientation of the object. Thus, because the object is vertical, the image must be vertical. Therefore, the image of the base of the object is on the optical axis directly above the image of the tip, as drawn in the figure.

For the concave mirror, the extended image in this case forms between the focal point and the center of curvature of the mirror. It is inverted with respect to the object, is a real image, and is smaller than the object. Were we to move the object closer to or farther from the mirror, the characteristics of the image would change. For example, we show, as a later exercise, that an object placed between a concave mirror and its focal point leads to a virtual image that is upright and larger than the object. For the convex mirror, the extended image forms between the focal point and the mirror. It is upright with respect to the object, is a virtual image, and is smaller than the object.

Summary of Ray-Tracing Rules

Ray tracing is very useful for mirrors. The rules for ray tracing are summarized here for reference:

- A ray travelling parallel to the optical axis of a spherical mirror is reflected along a line that goes through the focal point of the mirror (ray 1 in **Figure 2.9**).

- A ray travelling along a line that goes through the focal point of a spherical mirror is reflected along a line parallel to the optical axis of the mirror (ray 2 in **Figure 2.9**).
- A ray travelling along a line that goes through the center of curvature of a spherical mirror is reflected back along the same line (ray 3 in **Figure 2.9**).
- A ray that strikes the vertex of a spherical mirror is reflected symmetrically about the optical axis of the mirror (ray 4 in **Figure 2.9**).

We use ray tracing to illustrate how images are formed by mirrors and to obtain numerical information about optical properties of the mirror. If we assume that a mirror is small compared with its radius of curvature, we can also use algebra and geometry to derive a mirror equation, which we do in the next section. Combining ray tracing with the mirror equation is a good way to analyze mirror systems.

Image Formation by Reflection—The Mirror Equation

For a plane mirror, we showed that the image formed has the same height and orientation as the object, and it is located at the same distance behind the mirror as the object is in front of the mirror. Although the situation is a bit more complicated for curved mirrors, using geometry leads to simple formulas relating the object and image distances to the focal lengths of concave and convex mirrors.

Consider the object *OP* shown in **Figure 2.10**. The center of curvature of the mirror is labeled *C* and is a distance *R* from the vertex of the mirror, as marked in the figure. The object and image distances are labeled d_o and d_i, and the object and image heights are labeled h_o and h_i, respectively. Because the angles ϕ and ϕ' are alternate interior angles, we know that they have the same magnitude. However, they must differ in sign if we measure angles from the optical axis, so $\phi = -\phi'$. An analogous scenario holds for the angles θ and θ'. The law of reflection tells us that they have the same magnitude, but their signs must differ if we measure angles from the optical axis. Thus, $\theta = -\theta'$. Taking the tangent of the angles θ and θ', and using the property that $\tan(-\theta) = -\tan\theta$, gives us

$$\left.\begin{aligned}\tan\theta &= \frac{h_o}{d_o}\\ \tan\theta' = -\tan\theta &= \frac{h_i}{d_i}\end{aligned}\right\}\frac{h_o}{d_o} = -\frac{h_i}{d_i} \text{ or } -\frac{h_o}{h_i} = \frac{d_o}{d_i}. \tag{2.3}$$

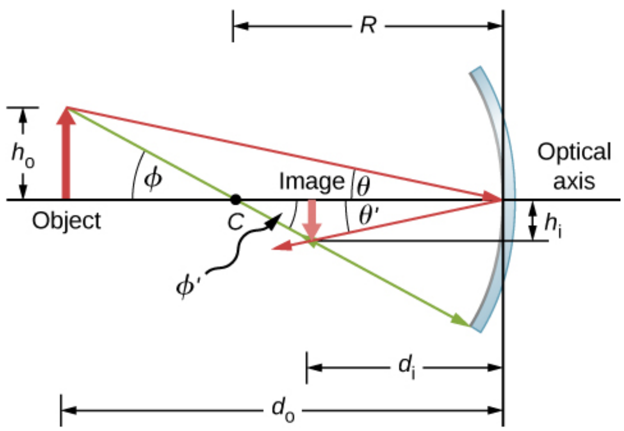


Figure 2.10 Image formed by a concave mirror.

Similarly, taking the tangent of ϕ and ϕ' gives

$$\left.\begin{aligned}\tan\phi &= \frac{h_o}{d_o - R}\\ \tan\phi' = -\tan\phi &= \frac{h_i}{R - d_i}\end{aligned}\right\}\frac{h_o}{d_o - R} = -\frac{h_i}{R - d_i} \text{ or } -\frac{h_o}{h_i} = \frac{d_o - R}{R - d_i}.$$

Combining these two results gives

$$\frac{d_o}{d_i} = \frac{d_o - R}{R - d_i}.$$

After a little algebra, this becomes

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{2}{R}. \tag{2.4}$$

No approximation is required for this result, so it is exact. However, as discussed above, in the small-angle approximation, the focal length of a spherical mirror is one-half the radius of curvature of the mirror, or $f = R/2$. Inserting this into **Equation 2.3** gives the *mirror equation*:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}. \tag{2.5}$$

The mirror equation relates the image and object distances to the focal distance and is valid only in the small-angle approximation. Although it was derived for a concave mirror, it also holds for convex mirrors (proving this is left as an exercise). We can extend the mirror equation to the case of a plane mirror by noting that a plane mirror has an infinite radius of curvature. This means the focal point is at infinity, so the mirror equation simplifies to

$$d_o = -d_i \tag{2.6}$$

which is the same as **Equation 2.1** obtained earlier.

Notice that we have been very careful with the signs in deriving the mirror equation. For a plane mirror, the image distance has the opposite sign of the object distance. Also, the real image formed by the concave mirror in **Figure 2.10** is on the opposite side of the optical axis with respect to the object. In this case, the image height should have the opposite sign of the object height. To keep track of the signs of the various quantities in the mirror equation, we now introduce a sign convention.

Sign convention for spherical mirrors

Using a consistent sign convention is very important in geometric optics. It assigns positive or negative values for the quantities that characterize an optical system. Understanding the sign convention allows you to describe an image without constructing a ray diagram. This text uses the following sign convention:

1. The focal length f is positive for concave mirrors and negative for convex mirrors.
2. The image distance d_i is positive for real images and negative for virtual images.

Notice that rule 1 means that the radius of curvature of a spherical mirror can be positive or negative. What does it mean to have a negative radius of curvature? This means simply that the radius of curvature for a convex mirror is defined to be negative.

Image magnification

Let's use the sign convention to further interpret the derivation of the mirror equation. In deriving this equation, we found that the object and image heights are related by

$$-\frac{h_o}{h_i} = \frac{d_o}{d_i}. \tag{2.7}$$

See **Equation 2.3**. Both the object and the image formed by the mirror in **Figure 2.10** are real, so the object and image distances are both positive. The highest point of the object is above the optical axis, so the object height is positive. The image, however, is below the optical axis, so the image height is negative. Thus, this sign convention is consistent with our derivation of the mirror equation.

Equation 2.7 in fact describes the **linear magnification** (often simply called "magnification") of the image in terms of the object and image distances. We thus define the dimensionless magnification m as follows:

$$m = \frac{h_i}{h_o}. \tag{2.8}$$

If m is positive, the image is upright, and if m is negative, the image is inverted. If $|m| > 1$, the image is larger than the object, and if $|m| < 1$, the image is smaller than the object. With this definition of magnification, we get the following relation between the vertical and horizontal object and image distances:

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}. \tag{2.9}$$

This is a very useful relation because it lets you obtain the magnification of the image from the object and image distances, which you can obtain from the mirror equation.

Example 2.1

Solar Electric Generating System

One of the solar technologies used today for generating electricity involves a device (called a parabolic trough or concentrating collector) that concentrates sunlight onto a blackened pipe that contains a fluid. This heated fluid is pumped to a heat exchanger, where the thermal energy is transferred to another system that is used to generate steam and eventually generates electricity through a conventional steam cycle. **Figure 2.11** shows such a working system in southern California. The real mirror is a parabolic cylinder with its focus located at the pipe; however, we can approximate the mirror as exactly one-quarter of a circular cylinder.

Figure 2.11 Parabolic trough collectors are used to generate electricity in southern California. (credit: "kjkolb"/Wikimedia Commons)

a. If we want the rays from the sun to focus at 40.0 cm from the mirror, what is the radius of the mirror?
b. What is the amount of sunlight concentrated onto the pipe, per meter of pipe length, assuming the insolation (incident solar radiation) is 900 W/m^2?
c. If the fluid-carrying pipe has a 2.00-cm diameter, what is the temperature increase of the fluid per meter of pipe over a period of 1 minute? Assume that all solar radiation incident on the reflector is absorbed by the pipe, and that the fluid is mineral oil.

Strategy

First identify the physical principles involved. Part (a) is related to the optics of spherical mirrors. Part (b) involves a little math, primarily geometry. Part (c) requires an understanding of heat and density.

Solution

a. The sun is the object, so the object distance is essentially infinity: $d_o = \infty$. The desired image distance is $d_i = 40.0\,\text{cm}$. We use the mirror equation to find the focal length of the mirror:

$$\begin{aligned}\frac{1}{d_o}+\frac{1}{d_i} &= \frac{1}{f}\\ f &= \left(\frac{1}{d_o}+\frac{1}{d_i}\right)^{-1}\\ &= \left(\frac{1}{\infty}+\frac{1}{40.0\text{ cm}}\right)^{-1}\\ &= 40.0\text{ cm}\end{aligned}$$

Thus, the radius of the mirror is $R = 2f = 80.0\text{ cm}$.

b. The insolation is 900 W/m^2. You must find the cross-sectional area A of the concave mirror, since the power delivered is 900 $\text{W/m}^2 \times A$. The mirror in this case is a quarter-section of a cylinder, so the area for a length L of the mirror is $A = \frac{1}{4}(2\pi R)L$. The area for a length of 1.00 m is then

$$A = \frac{\pi}{2}R(1.00\text{ m}) = \frac{(3.14)}{2}(0.800\text{ m})(1.00\text{ m}) = 1.26\text{ m}^2.$$

The insolation on the 1.00-m length of pipe is then

$$\left(9.00\times 10^2\frac{\text{W}}{\text{m}^2}\right)\left(1.26\text{ m}^2\right) = 1130\text{ W}.$$

c. The increase in temperature is given by $Q = mc\Delta T$. The mass m of the mineral oil in the one-meter section of pipe is

$$\begin{aligned}m &= \rho V = \rho\pi\left(\frac{d}{2}\right)^2(1.00\text{ m})\\ &= \left(8.00\times 10^2\text{ kg/m}^3\right)(3.14)(0.0100\text{ m})^2(1.00\text{ m})\\ &= 0.251\text{ kg}\end{aligned}$$

Therefore, the increase in temperature in one minute is

$$\begin{aligned}\Delta T &= Q/mc\\ &= \frac{(1130\text{ W})(60.0\text{ s})}{(0.251\text{ kg})(1670\text{ J}\cdot\text{kg/°C})}\\ &= 162\text{°C}\end{aligned}$$

Significance

An array of such pipes in the California desert can provide a thermal output of 250 MW on a sunny day, with fluids reaching temperatures as high as 400°C. We are considering only one meter of pipe here and ignoring heat losses along the pipe.

Example 2.2

Image in a Convex Mirror

A keratometer is a device used to measure the curvature of the cornea of the eye, particularly for fitting contact lenses. Light is reflected from the cornea, which acts like a convex mirror, and the keratometer measures the magnification of the image. The smaller the magnification, the smaller the radius of curvature of the cornea. If the light source is 12 cm from the cornea and the image magnification is 0.032, what is the radius of curvature of the cornea?

Strategy

If you find the focal length of the convex mirror formed by the cornea, then you know its radius of curvature (it's twice the focal length). The object distance is $d_o = 12\,\text{cm}$ and the magnification is $m = 0.032$. First find the image distance d_i and then solve for the focal length *f*.

Solution

Start with the equation for magnification, $m = -d_i/d_o$. Solving for d_i and inserting the given values yields

$$d_i = -md_o = -(0.032)(12\,\text{cm}) = -0.384\,\text{cm}$$

where we retained an extra significant figure because this is an intermediate step in the calculation. Solve the mirror equation for the focal length *f* and insert the known values for the object and image distances. The result is

$$\begin{aligned}
\frac{1}{d_o} + \frac{1}{d_i} &= \frac{1}{f} \\
f &= \left(\frac{1}{d_o} + \frac{1}{d_i}\right)^{-1} \\
&= \left(\frac{1}{12\,\text{cm}} + \frac{1}{-0.384\,\text{cm}}\right)^{-1} \\
&= -0.40\,\text{cm}
\end{aligned}$$

The radius of curvature is twice the focal length, so

$$R = 2f = -0.80\,\text{cm}$$

Significance

The focal length is negative, so the focus is virtual, as expected for a concave mirror and a real object. The radius of curvature found here is reasonable for a cornea. The distance from cornea to retina in an adult eye is about 2.0 cm. In practice, corneas may not be spherical, which complicates the job of fitting contact lenses. Note that the image distance here is negative, consistent with the fact that the image is behind the mirror. Thus, the image is virtual because no rays actually pass through it. In the problems and exercises, you will show that, for a fixed object distance, a smaller radius of curvature corresponds to a smaller the magnification.

Problem-Solving Strategy: Spherical Mirrors

Step 1. First make sure that image formation by a spherical mirror is involved.

Step 2. Determine whether ray tracing, the mirror equation, or both are required. A sketch is very useful even if ray tracing is not specifically required by the problem. Write symbols and known values on the sketch.

Step 3. Identify exactly what needs to be determined in the problem (identify the unknowns).

Step 4. Make a list of what is given or can be inferred from the problem as stated (identify the knowns).

Step 5. If ray tracing is required, use the ray-tracing rules listed near the beginning of this section.

Step 6. Most quantitative problems require using the mirror equation. Use the examples as guides for using the mirror equation.

Step 7. Check to see whether the answer makes sense. Do the signs of object distance, image distance, and focal length correspond with what is expected from ray tracing? Is the sign of the magnification correct? Are the object and image distances reasonable?

Departure from the Small-Angle Approximation

The small-angle approximation is a cornerstone of the above discussion of image formation by a spherical mirror. When this approximation is violated, then the image created by a spherical mirror becomes distorted. Such distortion is called **aberration**. Here we briefly discuss two specific types of aberrations: spherical aberration and coma.

Spherical aberration

Consider a broad beam of parallel rays impinging on a spherical mirror, as shown in **Figure 2.12**.

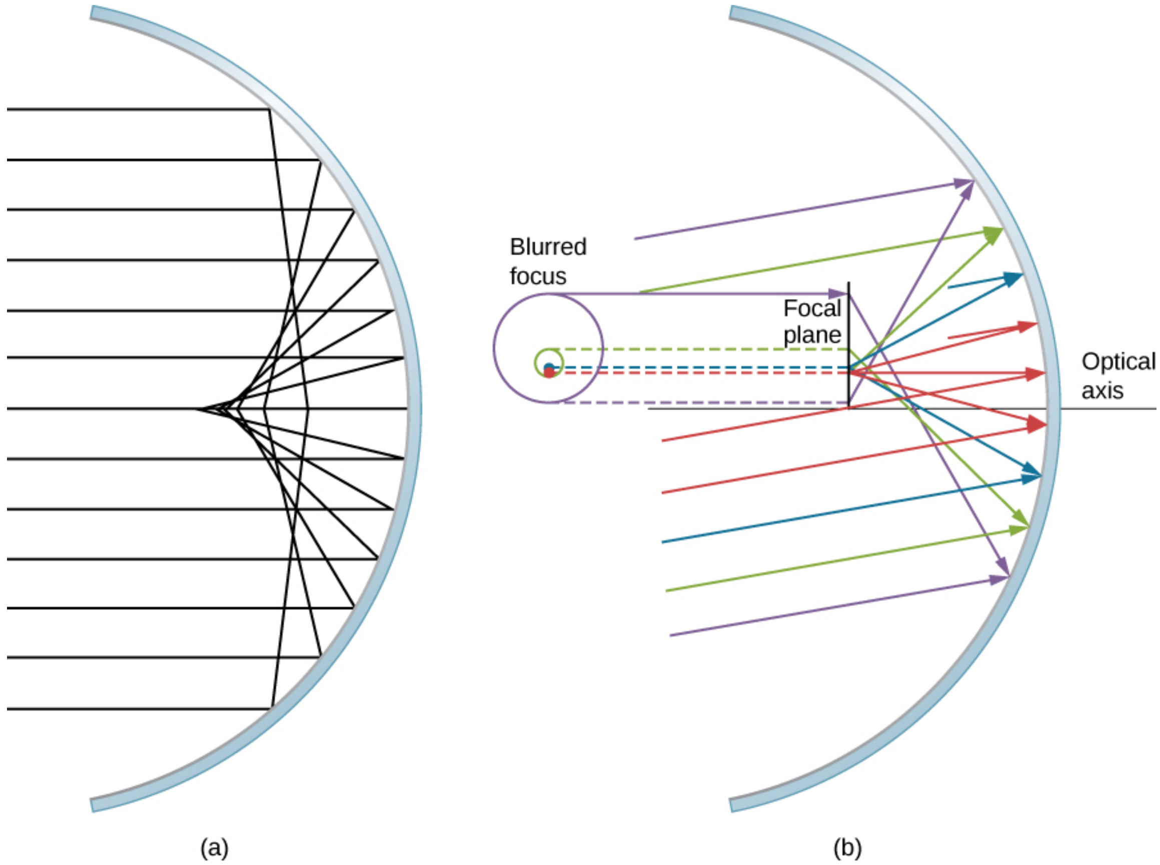


Figure 2.12 (a) With spherical aberration, the rays that are farther from the optical axis and the rays that are closer to the optical axis are focused at different points. Notice that the aberration gets worse for rays farther from the optical axis. (b) For comatic aberration, parallel rays that are not parallel to the optical axis are focused at different heights and at different focal lengths, so the image contains a "tail" like a comet (which is "coma" in Latin). Note that the colored rays are only to facilitate viewing; the colors do not indicate the color of the light.

The farther from the optical axis the rays strike, the worse the spherical mirror approximates a parabolic mirror. Thus, these rays are not focused at the same point as rays that are near the optical axis, as shown in the figure. Because of **spherical aberration**, the image of an extended object in a spherical mirror will be blurred. Spherical aberrations are characteristic of the mirrors and lenses that we consider in the following section of this chapter (more sophisticated mirrors and lenses are needed to eliminate spherical aberrations).

Coma or comatic aberration

Coma is similar to spherical aberration, but arises when the incoming rays are not parallel to the optical axis, as shown in part (b) of **Figure 2.12**. Recall that the small-angle approximation holds for spherical mirrors that are small compared to their radius. In this case, spherical mirrors are good approximations of parabolic mirrors. Parabolic mirrors focus all rays that are parallel to the optical axis at the focal point. However, parallel rays that are *not* parallel to the optical axis are focused at different heights and at different focal lengths, as show in part (b) of **Figure 2.12**. Because a spherical mirror is symmetric about the optical axis, the various colored rays in this figure create circles of the corresponding color on the focal plane.

Although a spherical mirror is shown in part (b) of **Figure 2.12**, comatic aberration occurs also for parabolic mirrors—it does not result from a breakdown in the small-angle approximation. Spherical aberration, however, occurs only for spherical mirrors and is a result of a breakdown in the small-angle approximation. We will discuss both coma and spherical aberration later in this chapter, in connection with telescopes.

2.3 | Images Formed by Refraction

Learning Objectives

By the end of this section, you will be able to:

- Describe image formation by a single refracting surface
- Determine the location of an image and calculate its properties by using a ray diagram
- Determine the location of an image and calculate its properties by using the equation for a single refracting surface

When rays of light propagate from one medium to another, these rays undergo refraction, which is when light waves are bent at the interface between two media. The refracting surface can form an image in a similar fashion to a reflecting surface, except that the law of refraction (Snell's law) is at the heart of the process instead of the law of reflection.

Refraction at a Plane Interface—Apparent Depth

If you look at a straight rod partially submerged in water, it appears to bend at the surface (**Figure 2.13**). The reason behind this curious effect is that the image of the rod inside the water forms a little closer to the surface than the actual position of the rod, so it does not line up with the part of the rod that is above the water. The same phenomenon explains why a fish in water appears to be closer to the surface than it actually is.

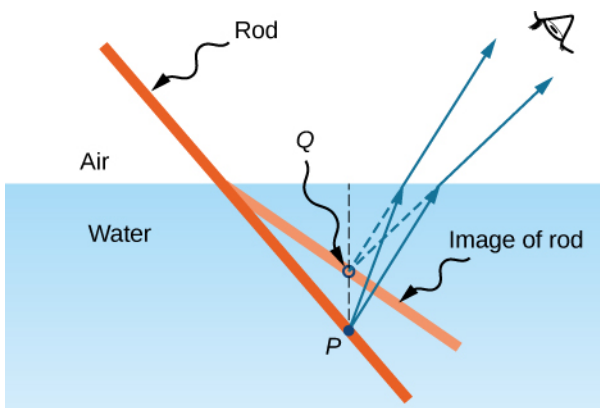


Figure 2.13 Bending of a rod at a water-air interface. Point *P* on the rod appears to be at point *Q*, which is where the image of point *P* forms due to refraction at the air-water interface.

To study image formation as a result of refraction, consider the following questions:

1. What happens to the rays of light when they enter or pass through a different medium?
2. Do the refracted rays originating from a single point meet at some point or diverge away from each other?

To be concrete, we consider a simple system consisting of two media separated by a plane interface (**Figure 2.14**). The object is in one medium and the observer is in the other. For instance, when you look at a fish from above the water surface, the fish is in medium 1 (the water) with refractive index 1.33, and your eye is in medium 2 (the air) with refractive index 1.00, and the surface of the water is the interface. The depth that you "see" is the image height h_i and is called the **apparent depth**. The actual depth of the fish is the object height h_o.

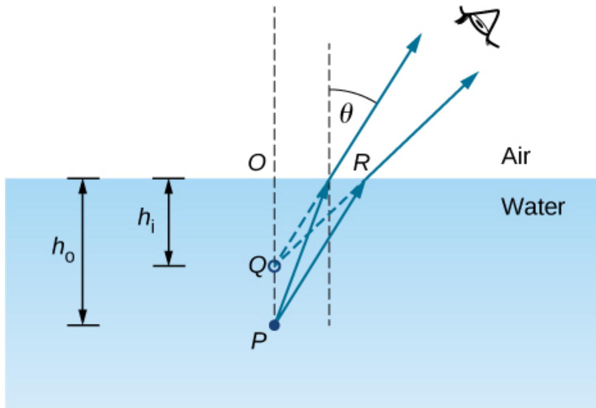


Figure 2.14 Apparent depth due to refraction. The real object at point *P* creates an image at point *Q*. The image is not at the same depth as the object, so the observer sees the image at an "apparent depth."

The apparent depth h_i depends on the angle at which you view the image. For a view from above (the so-called "normal" view), we can approximate the refraction angle θ to be small, and replace sin θ in Snell's law by tan θ. With this approximation, you can use the triangles ΔOPR and ΔOQR to show that the apparent depth is given by

$$h_\text{i} = \left(\frac{n_2}{n_1}\right)h_\text{o}. \tag{2.10}$$

The derivation of this result is left as an exercise. Thus, a fish appears at 3/4 of the real depth when viewed from above.

Refraction at a Spherical Interface

Spherical shapes play an important role in optics primarily because high-quality spherical shapes are far easier to manufacture than other curved surfaces. To study refraction at a single spherical surface, we assume that the medium with the spherical surface at one end continues indefinitely (a "semi-infinite" medium).

Refraction at a convex surface

Consider a point source of light at point *P* in front of a convex surface made of glass (see **Figure 2.15**). Let *R* be the radius of curvature, n_1 be the refractive index of the medium in which object point *P* is located, and n_2 be the refractive index of the medium with the spherical surface. We want to know what happens as a result of refraction at this interface.

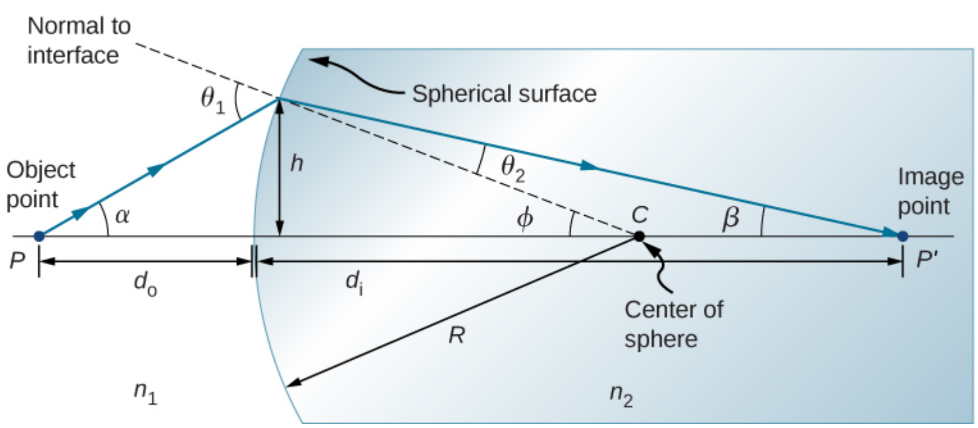


Figure 2.15 Refraction at a convex surface $(n_2 > n_1)$.

Because of the symmetry involved, it is sufficient to examine rays in only one plane. The figure shows a ray of light that starts at the object point *P*, refracts at the interface, and goes through the image point P'. We derive a formula relating the object distance d_o, the image distance d_i, and the radius of curvature *R*.

Applying Snell's law to the ray emanating from point *P* gives $n_1 \sin\theta_1 = n_2 \sin\theta_2$. We work in the small-angle approximation, so $\sin\theta \approx \theta$ and Snell's law then takes the form

$$n_1\theta_1 \approx n_2\theta_2.$$

From the geometry of the figure, we see that

$$\theta_1 = \alpha + \phi, \quad \theta_2 = \phi - \beta.$$

Inserting these expressions into Snell's law gives

$$n_1(\alpha + \phi) \approx n_2(\phi - \beta).$$

Using the diagram, we calculate the tangent of the angles α, β, and ϕ:

$$\tan\alpha \approx \frac{h}{d_o}, \quad \tan\beta \approx \frac{h}{d_i}, \quad \tan\phi \approx \frac{h}{R}.$$

Again using the small-angle approximation, we find that $\tan\theta \approx \theta$, so the above relationships become

$$\alpha \approx \frac{h}{d_o}, \quad \beta \approx \frac{h}{d_i}, \quad \phi \approx \frac{h}{R}.$$

Putting these angles into Snell's law gives

$$n_1\left(\frac{h}{d_o} + \frac{h}{R}\right) = n_2\left(\frac{h}{R} - \frac{h}{d_i}\right).$$

We can write this more conveniently as

$$\frac{n_1}{d_o} + \frac{n_2}{d_i} = \frac{n_2 - n_1}{R}. \tag{2.11}$$

If the object is placed at a special point called the **first focus**, or the **object focus** F_1, then the image is formed at infinity, as shown in part (a) of Figure 2.16.

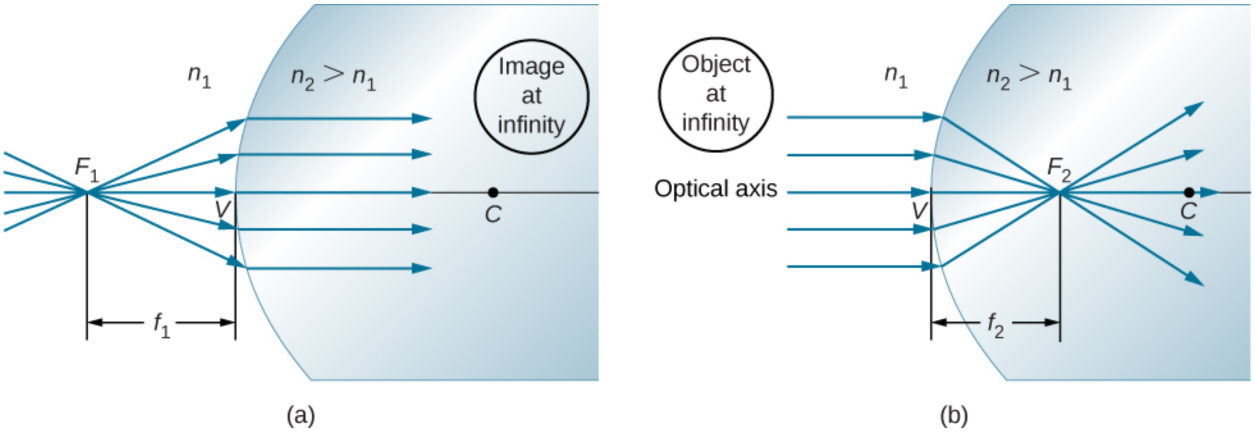


Figure 2.16 (a) First focus (called the "object focus") for refraction at a convex surface. (b) Second focus (called "image focus") for refraction at a convex surface.

We can find the location f_1 of the first focus F_1 by setting $d_i = \infty$ in the preceding equation.

$$\frac{n_1}{f_1} + \frac{n_2}{\infty} = \frac{n_2 - n_1}{R} \tag{2.12}$$

$$f_1 = \frac{n_1 R}{n_2 - n_1} \tag{2.13}$$

Similarly, we can define a **second focus** or **image focus** F_2 where the image is formed for an object that is far away [part (b)]. The location of the second focus F_2 is obtained from **Equation 2.11** by setting $d_o = \infty$:

$$\frac{n_1}{\infty} + \frac{n_2}{f_2} = \frac{n_2 - n_1}{R}$$

$$f_2 = \frac{n_2 R}{n_2 - n_1}.$$

Note that the object focus is at a different distance from the vertex than the image focus because $n_1 \neq n_2$.

Sign convention for single refracting surfaces

Although we derived this equation for refraction at a convex surface, the same expression holds for a concave surface, provided we use the following sign convention:

1. $R > 0$ if surface is convex toward object; otherwise, $R < 0$.
2. $d_i > 0$ if image is real and on opposite side from the object; otherwise, $d_i < 0$.

2.4 | Thin Lenses

Learning Objectives

By the end of this section, you will be able to:

- Use ray diagrams to locate and describe the image formed by a lens
- Employ the thin-lens equation to describe and locate the image formed by a lens

Lenses are found in a huge array of optical instruments, ranging from a simple magnifying glass to a camera’s zoom lens to the eye itself. In this section, we use the Snell’s law to explore the properties of lenses and how they form images.

The word “lens” derives from the Latin word for a lentil bean, the shape of which is similar to a convex lens. However, not all lenses have the same shape. **Figure 2.17** shows a variety of different lens shapes. The vocabulary used to describe lenses is the same as that used for spherical mirrors: The axis of symmetry of a lens is called the optical axis, where this axis intersects the lens surface is called the vertex of the lens, and so forth.

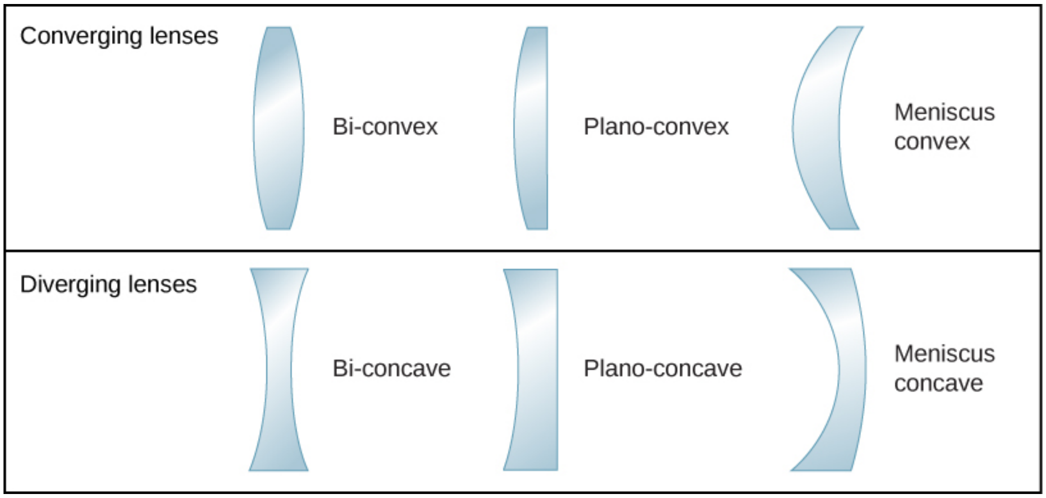


Figure 2.17 Various types of lenses: Note that a converging lens has a thicker “waist,” whereas a diverging lens has a thinner waist.

A **convex** or **converging lens** is shaped so that all light rays that enter it parallel to its optical axis intersect (or focus) at a single point on the optical axis on the opposite side of the lens, as shown in part (a) of **Figure 2.18**. Likewise, a **concave** or **diverging lens** is shaped so that all rays that enter it parallel to its optical axis diverge, as shown in part (b). To understand more precisely how a lens manipulates light, look closely at the top ray that goes through the converging lens in part (a). Because the index of refraction of the lens is greater than that of air, Snell's law tells us that the ray is bent toward the perpendicular to the interface as it enters the lens. Likewise, when the ray exits the lens, it is bent away from the perpendicular. The same reasoning applies to the diverging lenses, as shown in part (b). The overall effect is that light rays are bent toward the optical axis for a converging lens and away from the optical axis for diverging lenses. For a converging lens, the point at which the rays cross is the focal point *F* of the lens. For a diverging lens, the point from which the rays appear to originate is the (virtual) focal point. The distance from the center of the lens to its focal point is the focal length *f* of the lens.

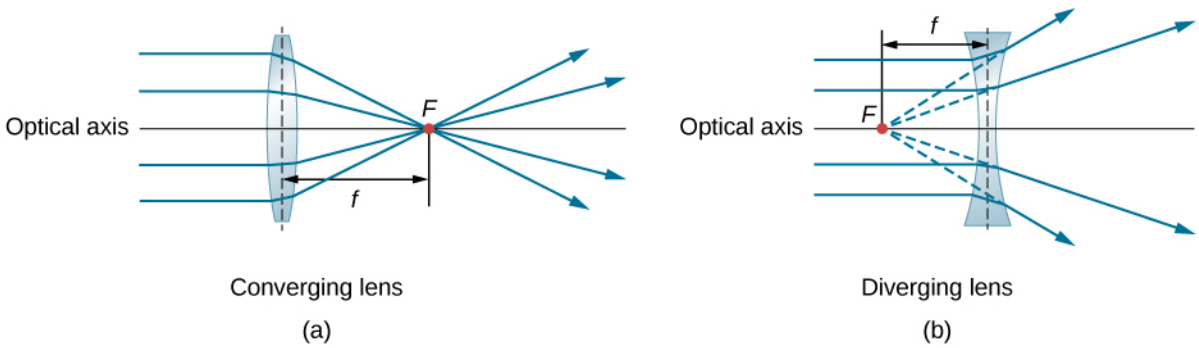


Figure 2.18 Rays of light entering (a) a converging lens and (b) a diverging lens, parallel to its axis, converge at its focal point *F*. The distance from the center of the lens to the focal point is the lens's focal length *f*. Note that the light rays are bent upon entering and exiting the lens, with the overall effect being to bend the rays toward the optical axis.

A lens is considered to be thin if its thickness *t* is much less than the radii of curvature of both surfaces, as shown in **Figure 2.19**. In this case, the rays may be considered to bend once at the center of the lens. For the case drawn in the figure, light ray 1 is parallel to the optical axis, so the outgoing ray is bent once at the center of the lens and goes through the focal point. Another important characteristic of thin lenses is that light rays that pass through the center of the lens are undeviated, as shown by light ray 2.

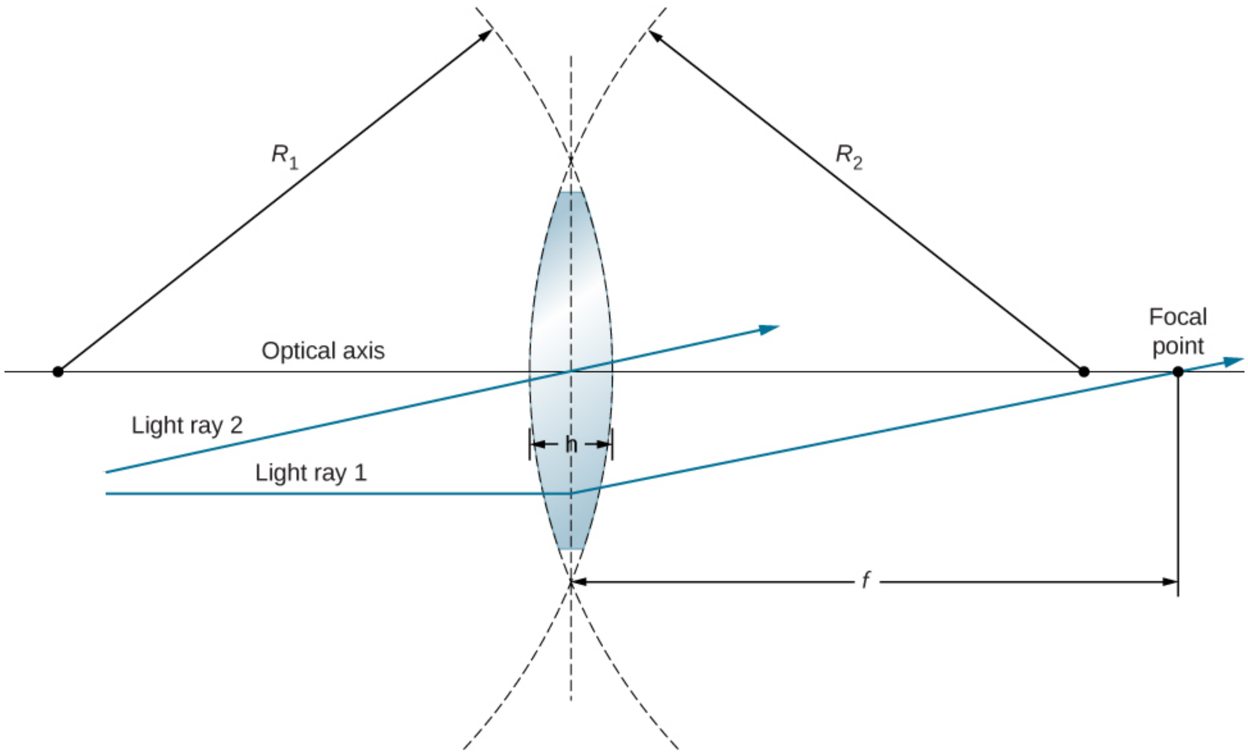


Figure 2.19 In the thin-lens approximation, the thickness *d* of the lens is much, much less than the radii R_1 and R_2 of curvature of the surfaces of the lens. Light rays are considered to bend at the center of the lens, such as light ray 1. Light ray 2 passes through the center of the lens and is undeviated in the thin-lens approximation.

As noted in the initial discussion of Snell's law, the paths of light rays are exactly reversible. This means that the direction of the arrows could be reversed for all of the rays in **Figure 2.18**. For example, if a point-light source is placed at the focal point of a convex lens, as shown in **Figure 2.20**, parallel light rays emerge from the other side.

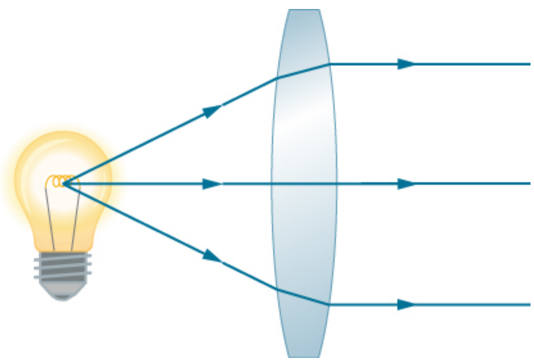


Figure 2.20 A small light source, like a light bulb filament, placed at the focal point of a convex lens results in parallel rays of light emerging from the other side. The paths are exactly the reverse of those shown in **Figure 2.18** in converging and diverging lenses. This technique is used in lighthouses and sometimes in traffic lights to produce a directional beam of light from a source that emits light in all directions.

Ray Tracing and Thin Lenses

Ray tracing is the technique of determining or following (tracing) the paths taken by light rays.

Ray tracing for thin lenses is very similar to the technique we used with spherical mirrors. As for mirrors, ray tracing can accurately describe the operation of a lens. The rules for ray tracing for thin lenses are similar to those of spherical mirrors:

1. A ray entering a converging lens parallel to the optical axis passes through the focal point on the other side of the

lens (ray 1 in part (a) of **Figure 2.21**). A ray entering a diverging lens parallel to the optical axis exits along the line that passes through the focal point on the *same* side of the lens (ray 1 in part (b) of the figure).

2. A ray passing through the center of either a converging or a diverging lens is not deviated (ray 2 in parts (a) and (b)).
3. For a converging lens, a ray that passes through the focal point exits the lens parallel to the optical axis (ray 3 in part (a)). For a diverging lens, a ray that approaches along the line that passes through the focal point on the opposite side exits the lens parallel to the axis (ray 3 in part (b)).

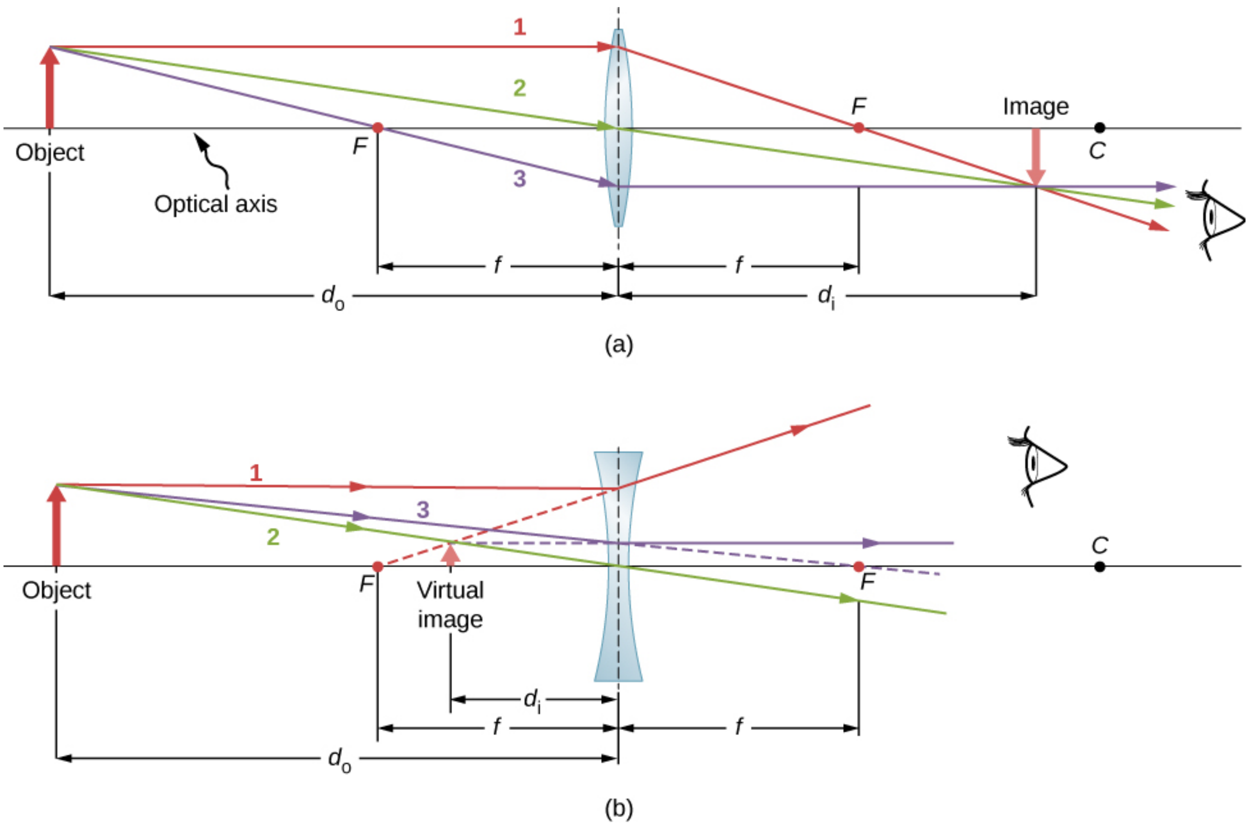


Figure 2.21 Thin lenses have the same focal lengths on either side. (a) Parallel light rays entering a converging lens from the right cross at its focal point on the left. (b) Parallel light rays entering a diverging lens from the right seem to come from the focal point on the right.

Thin lenses work quite well for monochromatic light (i.e., light of a single wavelength). However, for light that contains several wavelengths (e.g., white light), the lenses work less well. The problem is that, as we learned in the previous chapter, the index of refraction of a material depends on the wavelength of light. This phenomenon is responsible for many colorful effects, such as rainbows. Unfortunately, this phenomenon also leads to aberrations in images formed by lenses. In particular, because the focal distance of the lens depends on the index of refraction, it also depends on the wavelength of the incident light. This means that light of different wavelengths will focus at different points, resulting is so-called “chromatic aberrations.” In particular, the edges of an image of a white object will become colored and blurred. Special lenses called doublets are capable of correcting chromatic aberrations. A doublet is formed by gluing together a converging lens and a diverging lens. The combined doublet lens produces significantly reduced chromatic aberrations.

Image Formation by Thin Lenses

We use ray tracing to investigate different types of images that can be created by a lens. In some circumstances, a lens forms a real image, such as when a movie projector casts an image onto a screen. In other cases, the image is a virtual image, which cannot be projected onto a screen. Where, for example, is the image formed by eyeglasses? We use ray tracing for thin lenses to illustrate how they form images, and then we develop equations to analyze quantitatively the properties of thin lenses.

Consider an object some distance away from a converging lens, as shown in **Figure 2.22**. To find the location and size of the image, we trace the paths of selected light rays originating from one point on the object, in this case, the tip of the arrow.

The figure shows three rays from many rays that emanate from the tip of the arrow. These three rays can be traced by using the ray-tracing rules given above.

- Ray 1 enters the lens parallel to the optical axis and passes through the focal point on the opposite side (rule 1).
- Ray 2 passes through the center of the lens and is not deviated (rule 2).
- Ray 3 passes through the focal point on its way to the lens and exits the lens parallel to the optical axis (rule 3).

The three rays cross at a single point on the opposite side of the lens. Thus, the image of the tip of the arrow is located at this point. All rays that come from the tip of the arrow and enter the lens are refracted and cross at the point shown.

After locating the image of the tip of the arrow, we need another point of the image to orient the entire image of the arrow. We chose to locate the image base of the arrow, which is on the optical axis. As explained in the section on spherical mirrors, the base will be on the optical axis just above the image of the tip of the arrow (due to the top-bottom symmetry of the lens). Thus, the image spans the optical axis to the (negative) height shown. Rays from another point on the arrow, such as the middle of the arrow, cross at another common point, thus filling in the rest of the image.

Although three rays are traced in this figure, only two are necessary to locate a point of the image. It is best to trace rays for which there are simple ray-tracing rules.

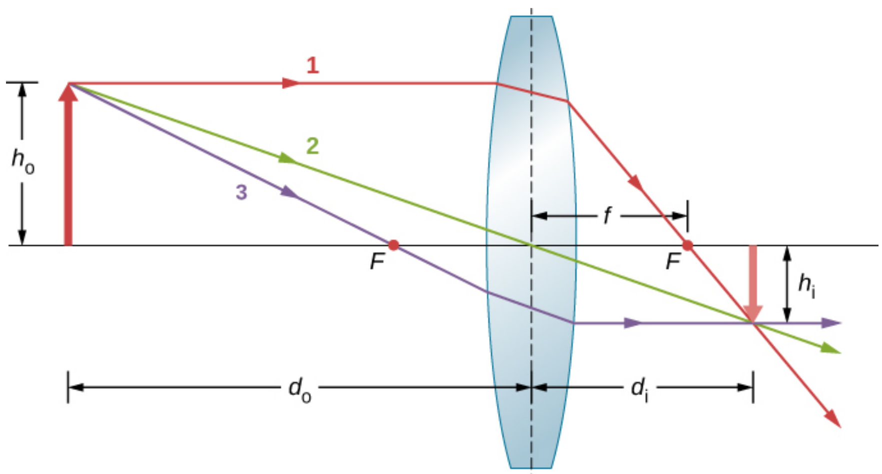


Figure 2.22 Ray tracing is used to locate the image formed by a lens. Rays originating from the same point on the object are traced—the three chosen rays each follow one of the rules for ray tracing, so that their paths are easy to determine. The image is located at the point where the rays cross. In this case, a real image—one that can be projected on a screen—is formed.

Several important distances appear in the figure. As for a mirror, we define d_o to be the object distance, or the distance of an object from the center of a lens. The image distance d_i is defined to be the distance of the image from the center of a lens. The height of the object and the height of the image are indicated by h_o and h_i, respectively. Images that appear upright relative to the object have positive heights, and those that are inverted have negative heights. By using the rules of ray tracing and making a scale drawing with paper and pencil, like that in **Figure 2.22**, we can accurately describe the location and size of an image. But the real benefit of ray tracing is in visualizing how images are formed in a variety of situations.

Oblique Parallel Rays and Focal Plane

We have seen that rays parallel to the optical axis are directed to the focal point of a converging lens. In the case of a diverging lens, they come out in a direction such that they appear to be coming from the focal point on the opposite side of the lens (i.e., the side from which parallel rays enter the lens). What happens to parallel rays that are not parallel to the optical axis (**Figure 2.23**)? In the case of a converging lens, these rays do not converge at the focal point. Instead, they come together on another point in the plane called the **focal plane**. The focal plane contains the focal point and is perpendicular to the optical axis. As shown in the figure, parallel rays focus where the ray through the center of the lens crosses the focal plane.

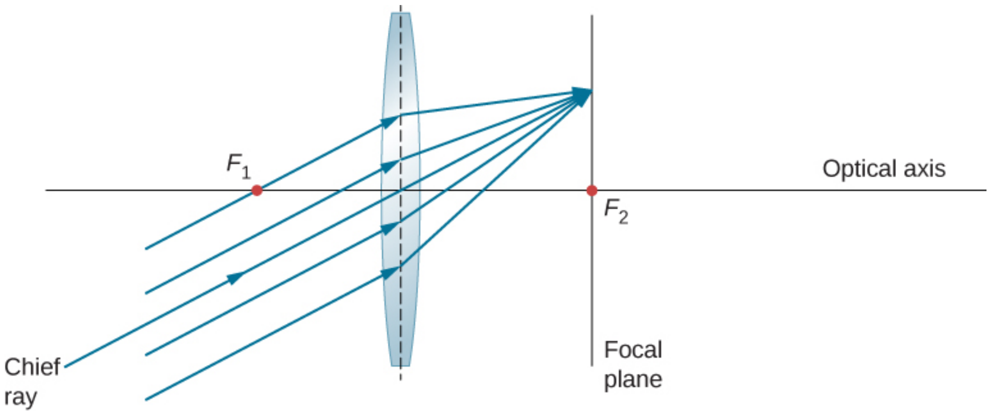


Figure 2.23 Parallel oblique rays focus on a point in a focal plane.

Thin-Lens Equation

Ray tracing allows us to get a qualitative picture of image formation. To obtain numeric information, we derive a pair of equations from a geometric analysis of ray tracing for thin lenses. These equations, called the thin-lens equation and the lens maker's equation, allow us to quantitatively analyze thin lenses.

Consider the thick bi-convex lens shown in **Figure 2.24**. The index of refraction of the surrounding medium is n_1 (if the lens is in air, then $n_1 = 1.00$) and that of the lens is n_2. The radii of curvatures of the two sides are R_1 and R_2. We wish to find a relation between the object distance d_o, the image distance d_i, and the parameters of the lens.

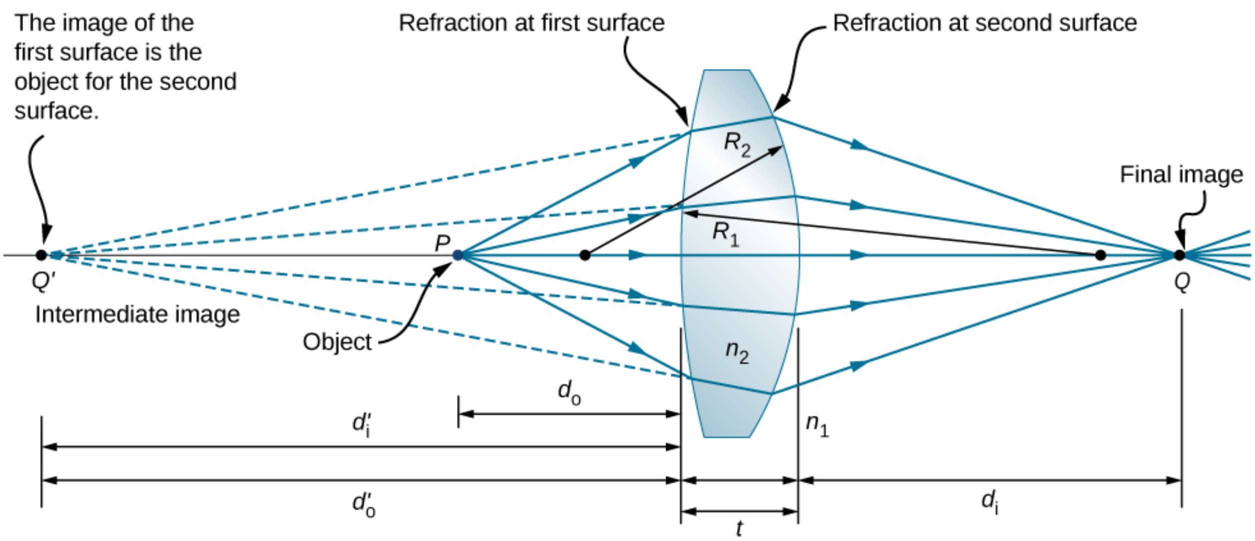


Figure 2.24 Figure for deriving the lens maker's equation. Here, *t* is the thickness of lens, n_1 is the index of refraction of the exterior medium, and n_2 is the index of refraction of the lens. We take the limit of $t \to 0$ to obtain the formula for a thin lens.

To derive the thin-lens equation, we consider the image formed by the first refracting surface (i.e., left surface) and then use this image as the object for the second refracting surface. In the figure, the image from the first refracting surface is Q', which is formed by extending backwards the rays from inside the lens (these rays result from refraction at the first surface). This is shown by the dashed lines in the figure. Notice that this image is virtual because no rays actually pass through the point Q'. To find the image distance d_i' corresponding to the image Q', we use **Equation 2.11**. In this case, the object distance is d_o, the image distance is d_i', and the radius of curvature is R_1. Inserting these into **Equation 2.3** gives

$$\frac{n_1}{d_o} + \frac{n_2}{d_i'} = \frac{n_2 - n_1}{R_1}. \tag{2.14}$$

The image is virtual and on the same side as the object, so $d_i' < 0$ and $d_o > 0$. The first surface is convex toward the object, so $R_1 > 0$.

To find the object distance for the object Q formed by refraction from the second interface, note that the role of the indices of refraction n_1 and n_2 are interchanged in **Equation 2.11**. In **Figure 2.24**, the rays originate in the medium with index n_2, whereas in **Figure 2.15**, the rays originate in the medium with index n_1. Thus, we must interchange n_1 and n_2 in **Equation 2.11**. In addition, by consulting again **Figure 2.24**, we see that the object distance is d'_o and the image distance is d_i. The radius of curvature is R_2 Inserting these quantities into **Equation 2.11** gives

$$\frac{n_2}{d'_o} + \frac{n_1}{d_i} = \frac{n_1 - n_2}{R_2}. \tag{2.15}$$

The image is real and on the opposite side from the object, so $d_i > 0$ and $d'_o > 0$. The second surface is convex away from the object, so $R_2 < 0$. **Equation 2.15** can be simplified by noting that $d'_o = |d'_i| + t$, where we have taken the absolute value because d'_i is a negative number, whereas both d'_o and t are positive. We can dispense with the absolute value if we negate d'_i, which gives $d'_o = -d'_i + t$. Inserting this into **Equation 2.15** gives

$$\frac{n_2}{-d'_i + t} + \frac{n_1}{d_i} = \frac{n_1 - n_2}{R_2}. \tag{2.16}$$

Summing **Equation 2.14** and **Equation 2.16** gives

$$\frac{n_1}{d_o} + \frac{n_1}{d_i} + \frac{n_2}{d'_i} + \frac{n_2}{-d'_i + t} = (n_2 - n_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right). \tag{2.17}$$

In the **thin-lens approximation**, we assume that the lens is very thin compared to the first image distance, or $t \ll d'_i$ (or, equivalently, $t \ll R_1$ and R_2). In this case, the third and fourth terms on the left-hand side of **Equation 2.17** cancel, leaving us with

$$\frac{n_1}{d_o} + \frac{n_1}{d_i} = (n_2 - n_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right).$$

Dividing by n_1 gives us finally

$$\frac{1}{d_o} + \frac{1}{d_i} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right). \tag{2.18}$$

The left-hand side looks suspiciously like the mirror equation that we derived above for spherical mirrors. As done for spherical mirrors, we can use ray tracing and geometry to show that, for a thin lens,

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \tag{2.19}$$

where f is the focal length of the thin lens (this derivation is left as an exercise). This is the thin-lens equation. The focal length of a thin lens is the same to the left and to the right of the lens. Combining **Equation 2.18** and **Equation 2.19** gives

$$\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \tag{2.20}$$

which is called the lens maker's equation. It shows that the focal length of a thin lens depends only of the radii of curvature and the index of refraction of the lens and that of the surrounding medium. For a lens in air, $n_1 = 1.0$ and $n_2 \equiv n$, so the lens maker's equation reduces to

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right). \quad (2.21)$$

Sign conventions for lenses

To properly use the thin-lens equation, the following sign conventions must be obeyed:

1. d_i is positive if the image is on the side opposite the object (i.e., real image); otherwise, d_i is negative (i.e., virtual image).
2. f is positive for a converging lens and negative for a diverging lens.
3. R is positive for a surface convex toward the object, and negative for a surface concave toward object.

Magnification

By using a finite-size object on the optical axis and ray tracing, you can show that the magnification m of an image is

$$m \equiv \frac{h_i}{h_o} = -\frac{d_i}{d_o} \quad (2.22)$$

(where the three lines mean "is defined as"). This is exactly the same equation as we obtained for mirrors (see **Equation 2.8**). If $m > 0$, then the image has the same vertical orientation as the object (called an "upright" image). If $m < 0$, then the image has the opposite vertical orientation as the object (called an "inverted" image).

Using the Thin-Lens Equation

The thin-lens equation and the lens maker's equation are broadly applicable to situations involving thin lenses. We explore many features of image formation in the following examples.

Consider a thin converging lens. Where does the image form and what type of image is formed as the object approaches the lens from infinity? This may be seen by using the thin-lens equation for a given focal length to plot the image distance as a function of object distance. In other words, we plot

$$d_i = \left(\frac{1}{f} - \frac{1}{d_o}\right)^{-1}$$

for a given value of f. For $f = 1$ cm, the result is shown in part (a) of **Figure 2.25**.

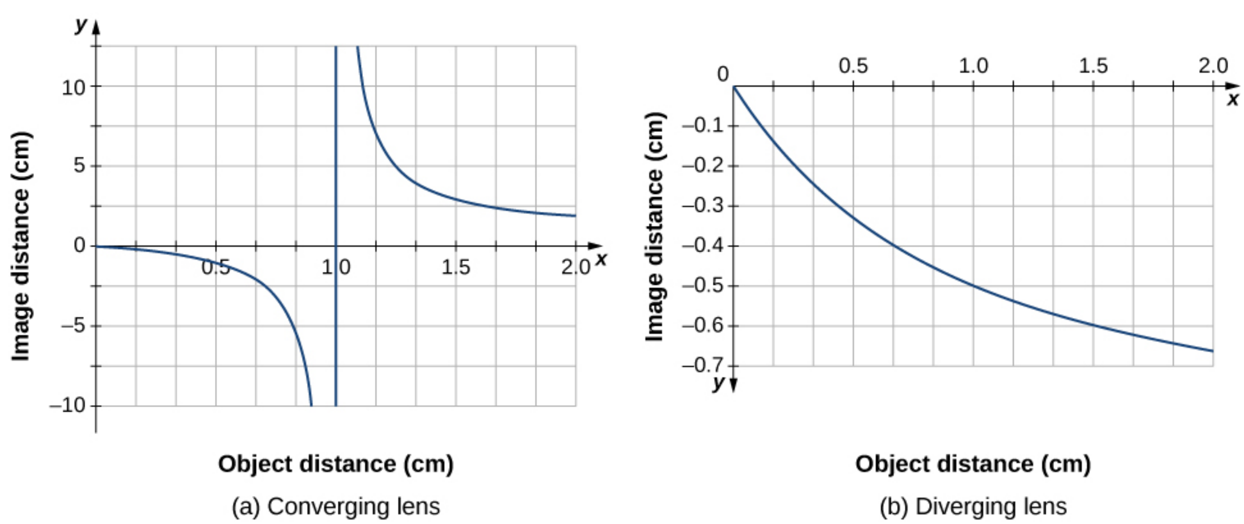


Figure 2.25 (a) Image distance for a thin converging lens with $f = 1.0$ cm as a function of object distance. (b) Same thing but for a diverging lens with $f = -1.0$ cm.

An object much farther than the focal length f from the lens should produce an image near the focal plane, because the second term on the right-hand side of the equation above becomes negligible compared to the first term, so we have $d_i \approx f$. This can be seen in the plot of part (a) of the figure, which shows that the image distance approaches asymptotically the focal length of 1 cm for larger object distances. As the object approaches the focal plane, the image distance diverges to positive infinity. This is expected because an object at the focal plane produces parallel rays that form an image at infinity (i.e., very far from the lens). When the object is farther than the focal length from the lens, the image distance is positive, so the image is real, on the opposite side of the lens from the object, and inverted (because $m = -d_i/d_o$). When the object is closer than the focal length from the lens, the image distance becomes negative, which means that the image is virtual, on the same side of the lens as the object, and upright.

For a thin diverging lens of focal length $f = -1.0\ \text{cm}$, a similar plot of image distance vs. object distance is shown in part (b). In this case, the image distance is negative for all positive object distances, which means that the image is virtual, on the same side of the lens as the object, and upright. These characteristics may also be seen by ray-tracing diagrams (see **Figure 2.26**).

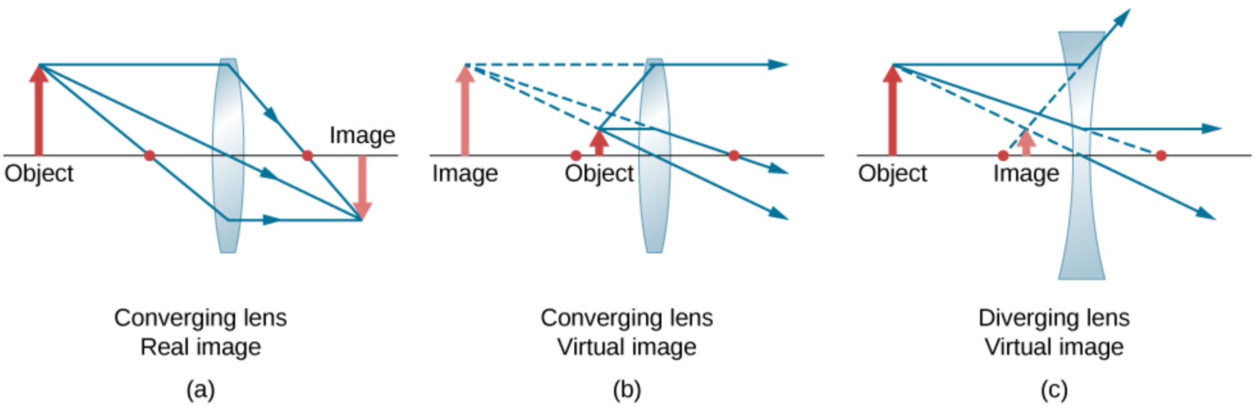


Figure 2.26 The red dots show the focal points of the lenses. (a) A real, inverted image formed from an object that is farther than the focal length from a converging lens. (b) A virtual, upright image formed from an object that is closer than a focal length from the lens. (c) A virtual, upright image formed from an object that is farther than a focal length from a diverging lens.

To see a concrete example of upright and inverted images, look at **Figure 2.27**, which shows images formed by converging lenses when the object (the person's face in this case) is place at different distances from the lens. In part (a) of the figure, the person's face is farther than one focal length from the lens, so the image is inverted. In part (b), the person's face is closer than one focal length from the lens, so the image is upright.

(a) (b)

Figure 2.27 (a) When a converging lens is held farther than one focal length from the man's face, an inverted image is formed. Note that the image is in focus but the face is not, because the image is much closer to the camera taking this photograph than the face. (b) An upright image of the man's face is produced when a converging lens is held at less than one focal length from his face. (credit a: modification of work by "DaMongMan"/Flickr; credit b: modification of work by Casey Fleser)

Work through the following examples to better understand how thin lenses work.

Problem-Solving Strategy: Lenses

Step 1. Determine whether ray tracing, the thin-lens equation, or both would be useful. Even if ray tracing is not used, a careful sketch is always very useful. Write symbols and values on the sketch.

Step 2. Identify what needs to be determined in the problem (identify the unknowns).

Step 3. Make a list of what is given or can be inferred from the problem (identify the knowns).

Step 4. If ray tracing is required, use the ray-tracing rules listed near the beginning of this section.

Step 5. Most quantitative problems require the use of the thin-lens equation and/or the lens maker's equation. Solve these for the unknowns and insert the given quantities or use both together to find two unknowns.

Step 7. Check to see if the answer is reasonable. Are the signs correct? Is the sketch or ray tracing consistent with the calculation?

Example 2.3

Using the Lens Maker's Equation

Find the radius of curvature of a biconcave lens symmetrically ground from a glass with index of refractive 1.55 so that its focal length in air is 20 cm (for a biconcave lens, both surfaces have the same radius of curvature).

Strategy

Use the thin-lens form of the lens maker's equation:

$$\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

where $R_1 < 0$ and $R_2 > 0$. Since we are making a symmetric biconcave lens, we have $|R_1| = |R_2|$.

Solution

We can determine the radius R of curvature from

$$\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\left(-\frac{2}{R}\right).$$

Solving for R and inserting $f = -20\text{ cm}$, $n_2 = 1.55$, and $n_1 = 1.00$ gives

$$R = -2f\left(\frac{n_2}{n_1} - 1\right) = -2(-20\text{ cm})\left(\frac{1.55}{1.00} - 1\right) = 22\text{ cm}.$$

Example 2.4

Converging Lens and Different Object Distances

Find the location, orientation, and magnification of the image for an 3.0 cm high object at each of the following positions in front of a convex lens of focal length 10.0 cm. (a) $d_o = 50.0\text{ cm}$, (b) $d_o = 5.00\text{ cm}$, and (c) $d_o = 20.0\text{ cm}$.

Strategy

We start with the thin-lens equation $\frac{1}{d_i} + \frac{1}{d_o} = \frac{1}{f}$. Solve this for the image distance d_i and insert the given object distance and focal length.

Solution

a. For $d_o = 50\text{ cm}$, $f = +10\text{ cm}$, this gives

$$\begin{aligned} d_i &= \left(\frac{1}{f} - \frac{1}{d_o}\right)^{-1} \\ &= \left(\frac{1}{10.0\text{ cm}} - \frac{1}{50.0\text{ cm}}\right)^{-1} \\ &= 12.5\text{ cm} \end{aligned}$$

The image is positive, so the image, is real, is on the opposite side of the lens from the object, and is 12.6 cm from the lens. To find the magnification and orientation of the image, use

$$m = -\frac{d_i}{d_o} = -\frac{12.5\text{ cm}}{50.0\text{ cm}} = -0.250.$$

The negative magnification means that the image is inverted. Since $|m| < 1$, the image is smaller than the object. The size of the image is given by

$$|h_i| = |m|h_o = (0.250)(3.0\text{ cm}) = 0.75\text{ cm}$$

b. For $d_o = 5.00\text{ cm}$, $f = +10.0\text{ cm}$

$$\begin{aligned} d_i &= \left(\frac{1}{f} - \frac{1}{d_o}\right)^{-1} \\ &= \left(\frac{1}{10.0\text{ cm}} - \frac{1}{5.00\text{ cm}}\right)^{-1} \\ &= -10.0\text{ cm} \end{aligned}$$

The image distance is negative, so the image is virtual, is on the same side of the lens as the object, and is 10 cm from the lens. The magnification and orientation of the image are found from

$$m = -\frac{d_i}{d_o} = -\frac{-10.0\text{ cm}}{5.00\text{ cm}} = +2.00.$$

The positive magnification means that the image is upright (i.e., it has the same orientation as the object). Since $|m| > 0$, the image is larger than the object. The size of the image is

$$|h_i| = |m|h_o = (2.00)(3.0\,\text{cm}) = 6.0\,\text{cm}.$$

c. For $d_o = 20\,\text{cm}, f = +10\,\text{cm}$

$$\begin{aligned} d_i &= \left(\frac{1}{f} - \frac{1}{d_o}\right)^{-1} \\ &= \left(\frac{1}{10.0\,\text{cm}} - \frac{1}{20.0\,\text{cm}}\right)^{-1} \\ &= 20.0\,\text{cm} \end{aligned}$$

The image distance is positive, so the image is real, is on the opposite side of the lens from the object, and is 20.0 cm from the lens. The magnification is

$$m = -\frac{d_i}{d_o} = -\frac{20.0\,\text{cm}}{20.0\,\text{cm}} = -1.00.$$

The negative magnification means that the image is inverted. Since $|m| = 1$, the image is the same size as the object.

When solving problems in geometric optics, we often need to combine ray tracing and the lens equations. The following example demonstrates this approach.

Example 2.5

Choosing the Focal Length and Type of Lens

To project an image of a light bulb on a screen 1.50 m away, you need to choose what type of lens to use (converging or diverging) and its focal length (**Figure 2.28**). The distance between the lens and the lightbulb is fixed at 0.75 m. Also, what is the magnification and orientation of the image?

Strategy

The image must be real, so you choose to use a converging lens. The focal length can be found by using the thin-lens equation and solving for the focal length. The object distance is $d_o = 0.75\,\text{m}$ and the image distance is $d_i = 1.5\,\text{m}$.

Solution

Solve the thin lens for the focal length and insert the desired object and image distances:

$$\begin{aligned} \frac{1}{d_o} + \frac{1}{d_i} &= \frac{1}{f} \\ f &= \left(\frac{1}{d_o} + \frac{1}{d_i}\right)^{-1} \\ &= \left(\frac{1}{0.75\,\text{m}} + \frac{1}{1.5\,\text{m}}\right)^{-1} \\ &= 0.50\,\text{m} \end{aligned}$$

The magnification is

$$m = -\frac{d_i}{d_o} = -\frac{1.5\,\text{m}}{0.75\,\text{m}} = -2.0.$$

Significance

The minus sign for the magnification means that the image is inverted. The focal length is positive, as expected for a converging lens. Ray tracing can be used to check the calculation (see **Figure 2.28**). As expected, the image is inverted, is real, and is larger than the object.

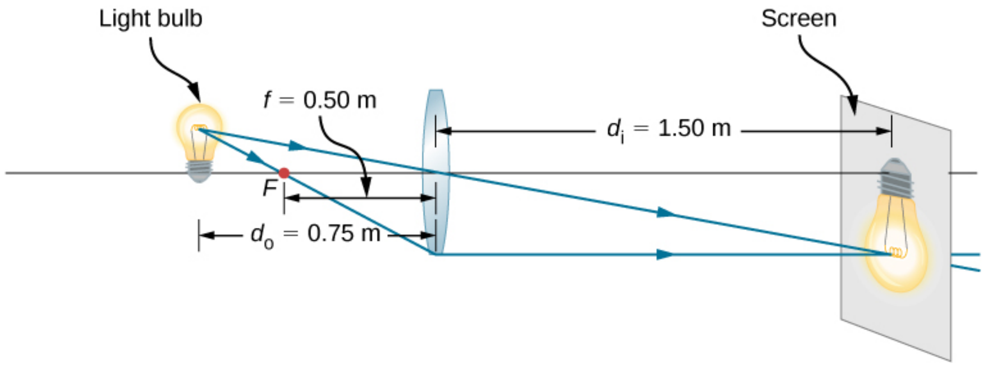


Figure 2.28 A light bulb placed 0.75 m from a lens having a 0.50-m focal length produces a real image on a screen, as discussed in the example. Ray tracing predicts the image location and size.

2.5 | The Eye

Learning Objectives

By the end of this section, you will be able to:

- Understand the basic physics of how images are formed by the human eye
- Recognize several conditions of impaired vision as well as the optics principles for treating these conditions

The human eye is perhaps the most interesting and important of all optical instruments. Our eyes perform a vast number of functions: They allow us to sense direction, movement, colors, and distance. In this section, we explore the geometric optics of the eye.

Physics of the Eye

The eye is remarkable in how it forms images and in the richness of detail and color it can detect. However, our eyes often need some correction to reach what is called “normal” vision. Actually, normal vision should be called “ideal” vision because nearly one-half of the human population requires some sort of eyesight correction, so requiring glasses is by no means “abnormal.” Image formation by our eyes and common vision correction can be analyzed with the optics discussed earlier in this chapter.

Figure 2.29 shows the basic anatomy of the eye. The cornea and lens form a system that, to a good approximation, acts as a single thin lens. For clear vision, a real image must be projected onto the light-sensitive retina, which lies a fixed distance from the lens. The flexible lens of the eye allows it to adjust the radius of curvature of the lens to produce an image on the retina for objects at different distances. The center of the image falls on the fovea, which has the greatest density of light receptors and the greatest acuity (sharpness) in the visual field. The variable opening (i.e., the pupil) of the eye, along with chemical adaptation, allows the eye to detect light intensities from the lowest observable to 10^{10} times greater (without damage). This is an incredible range of detection. Processing of visual nerve impulses begins with interconnections in the retina and continues in the brain. The optic nerve conveys the signals received by the eye to the brain.

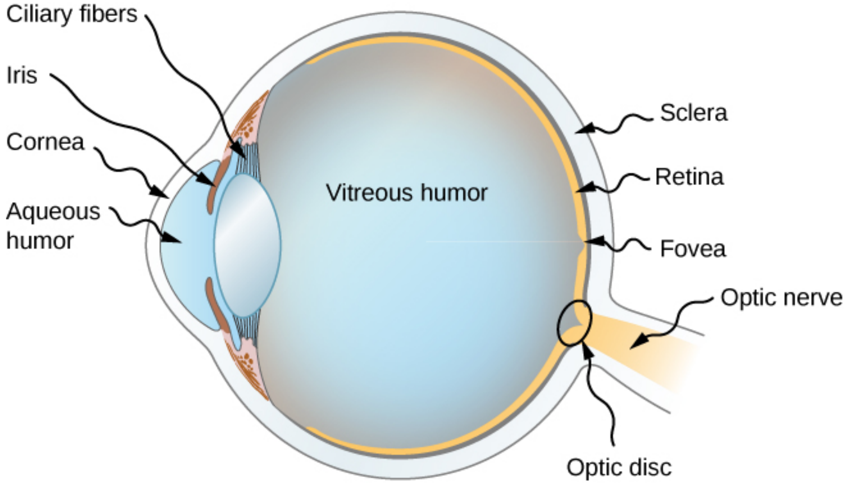


Figure 2.29 The cornea and lens of the eye act together to form a real image on the light-sensing retina, which has its densest concentration of receptors in the fovea and a blind spot over the optic nerve. The radius of curvature of the lens of an eye is adjustable to form an image on the retina for different object distances. Layers of tissues with varying indices of refraction in the lens are shown here. However, they have been omitted from other pictures for clarity.

The indices of refraction in the eye are crucial to its ability to form images. **Table 2.1** lists the indices of refraction relevant to the eye. The biggest change in the index of refraction, which is where the light rays are most bent, occurs at the air-cornea interface rather than at the aqueous humor-lens interface. The ray diagram in **Figure 2.30** shows image formation by the cornea and lens of the eye. The cornea, which is itself a converging lens with a focal length of approximately 2.3 cm, provides most of the focusing power of the eye. The lens, which is a converging lens with a focal length of about 6.4 cm, provides the finer focus needed to produce a clear image on the retina. The cornea and lens can be treated as a single thin lens, even though the light rays pass through several layers of material (such as cornea, aqueous humor, several layers in the lens, and vitreous humor), changing direction at each interface. The image formed is much like the one produced by a single convex lens (i.e., a real, inverted image). Although images formed in the eye are inverted, the brain inverts them once more to make them seem upright.

Material	Index of Refraction
Water	1.33
Air	1.0
Cornea	1.38
Aqueous humor	1.34
Lens	1.41*
Vitreous humor	1.34

Table 2.1 Refractive Indices Relevant to the Eye *This is an average value. The actual index of refraction varies throughout the lens and is greatest in center of the lens.

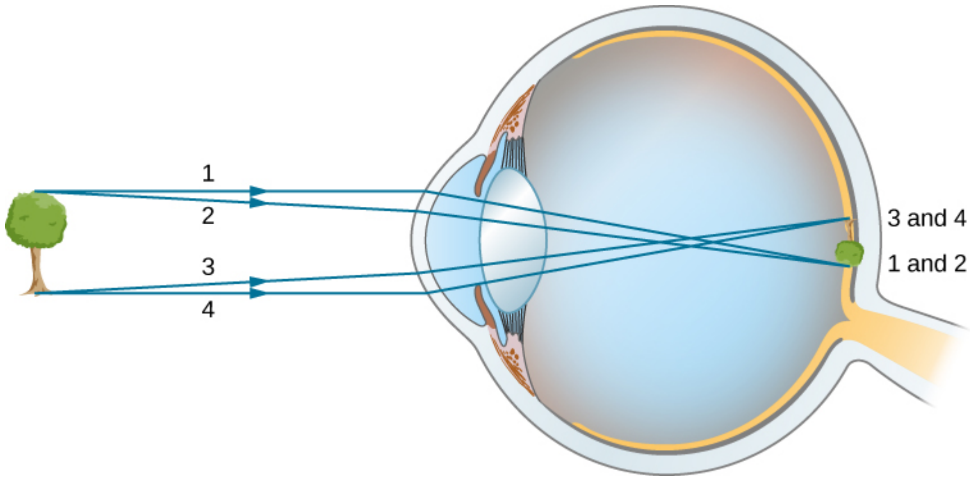


Figure 2.30 In the human eye, an image forms on the retina. Rays from the top and bottom of the object are traced to show how a real, inverted image is produced on the retina. The distance to the object is not to scale.

As noted, the image must fall precisely on the retina to produce clear vision—that is, the image distance d_i must equal the lens-to-retina distance. Because the lens-to-retina distance does not change, the image distance d_i must be the same for objects at all distances. The ciliary muscles adjust the shape of the eye lens for focusing on nearby or far objects. By changing the shape of the eye lens, the eye changes the focal length of the lens. This mechanism of the eye is called **accommodation**.

The nearest point an object can be placed so that the eye can form a clear image on the retina is called the **near point** of the eye. Similarly, the **far point** is the farthest distance at which an object is clearly visible. A person with normal vision can see objects clearly at distances ranging from 25 cm to essentially infinity. The near point increases with age, becoming several meters for some older people. In this text, we consider the near point to be 25 cm.

We can use the thin-lens equations to quantitatively examine image formation by the eye. First, we define the **optical power** of a lens as

$$P = \frac{1}{f} \tag{2.23}$$

with the focal length *f* given in meters. The units of optical power are called "diopters" (D). That is, $1\text{ D} = \frac{1}{\text{m}}$, or 1 m^{-1}.

Optometrists prescribe common eyeglasses and contact lenses in units of diopters. With this definition of optical power, we can rewrite the thin-lens equations as

$$P = \frac{1}{d_o} + \frac{1}{d_i}. \tag{2.24}$$

Working with optical power is convenient because, for two or more lenses close together, the effective optical power of the lens system is approximately the sum of the optical power of the individual lenses:

$$P_{\text{total}} = P_{\text{lens 1}} + P_{\text{lens 2}} + P_{\text{lens 3}} + \cdots \tag{2.25}$$

Example 2.6

Effective Focal Length of the Eye

The cornea and eye lens have focal lengths of 2.3 and 6.4 cm, respectively. Find the net focal length and optical power of the eye.

Strategy

The optical powers of the closely spaced lenses add, so $P_{\text{eye}} = P_{\text{cornea}} + P_{\text{lens}}$.

Solution

Writing the equation for power in terms of the focal lengths gives

$$\frac{1}{f_{\text{eye}}} = \frac{1}{f_{\text{cornea}}} + \frac{1}{f_{\text{lens}}} = \frac{1}{2.3\text{ cm}} + \frac{1}{6.4\text{ cm}}.$$

Hence, the focal length of the eye (cornea and lens together) is

$$f_{\text{eye}} = 1.69\text{ cm}.$$

The optical power of the eye is

$$P_{\text{eye}} = \frac{1}{f_{\text{eye}}} = \frac{1}{0.0169\text{ m}} = 59\text{ D}.$$

For clear vision, the image distance d_{i} must equal the lens-to-retina distance. Normal vision is possible for objects at distances $d_{\text{o}} = 25\text{ cm}$ to infinity. The following example shows how to calculate the image distance for an object placed at the near point of the eye.

Example 2.7

Image of an object placed at the near point

The net focal length of a particular human eye is 1.7 cm. An object is placed at the near point of the eye. How far behind the lens is a focused image formed?

Strategy

The near point is 25 cm from the eye, so the object distance is $d_{\text{o}} = 25\text{ cm}$. We determine the image distance from the lens equation:

$$\frac{1}{d_{\text{i}}} = \frac{1}{f} - \frac{1}{d_{\text{o}}}.$$

Solution

$$\begin{aligned} d_{\text{i}} &= \left(\frac{1}{f} - \frac{1}{d_{\text{o}}}\right)^{-1} \\ &= \left(\frac{1}{1.7\text{ cm}} - \frac{1}{25\text{ cm}}\right)^{-1} \\ &= 1.8\text{ cm} \end{aligned}$$

Therefore, the image is formed 1.8 cm behind the lens.

Significance

From the magnification formula, we find $m = -\frac{1.8\text{ cm}}{25\text{ cm}} = -0.073$. Since $m < 0$, the image is inverted in orientation with respect to the object. From the absolute value of *m* we see that the image is much smaller than the object; in fact, it is only 7% of the size of the object.

Vision Correction

The need for some type of vision correction is very common. Typical vision defects are easy to understand with geometric optics, and some are simple to correct. Figure 2.31 illustrates two common vision defects. **Nearsightedness**, or **myopia**, is the ability to see near objects, whereas distant objects are blurry. The eye overconverges the nearly parallel rays from a distant object, and the rays cross in front of the retina. More divergent rays from a close object are converged on the retina for a clear image. The distance to the farthest object that can be seen clearly is called the far point of the eye (normally the far point is at infinity). **Farsightedness**, or **hyperopia**, is the ability to see far objects clearly, whereas near objects are blurry. A farsighted eye does not sufficiently converge the rays from a near object to make the rays meet on the retina.

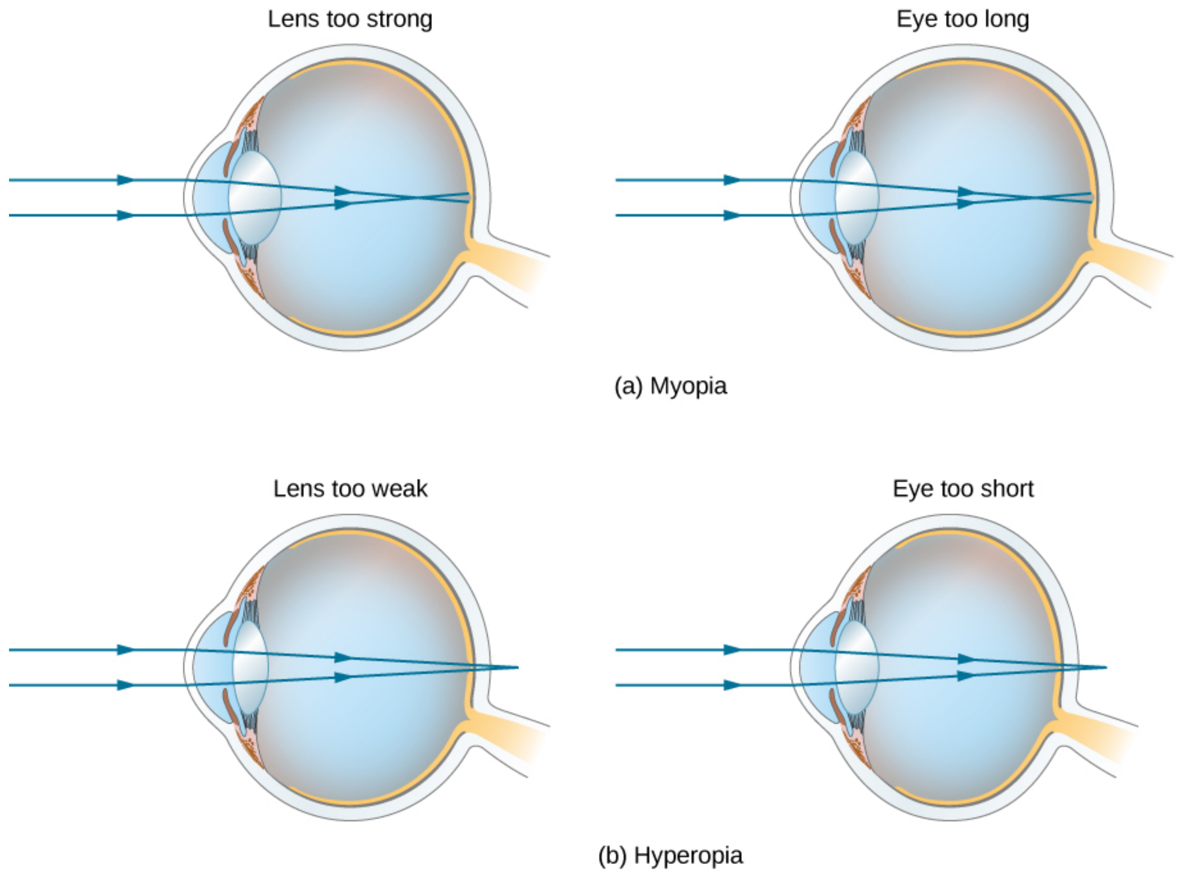


Figure 2.31 (a) The nearsighted (myopic) eye converges rays from a distant object in front of the retina, so they have diverged when they strike the retina, producing a blurry image. An eye lens that is too powerful can cause nearsightedness, or the eye may be too long. (b) The farsighted (hyperopic) eye is unable to converge the rays from a close object on the retina, producing blurry near-field vision. An eye lens with insufficient optical power or an eye that is too short can cause farsightedness.

Since the nearsighted eye overconverges light rays, the correction for nearsightedness consists of placing a diverging eyeglass lens in front of the eye, as shown in Figure 2.32. This reduces the optical power of an eye that is too powerful (recall that the focal length of a diverging lens is negative, so its optical power is negative). Another way to understand this correction is that a diverging lens will cause the incoming rays to diverge more to compensate for the excessive convergence caused by the lens system of the eye. The image produced by the diverging eyeglass lens serves as the (optical) object for the eye, and because the eye cannot focus on objects beyond its far point, the diverging lens must form an image of distant (physical) objects at a point that is closer than the far point.

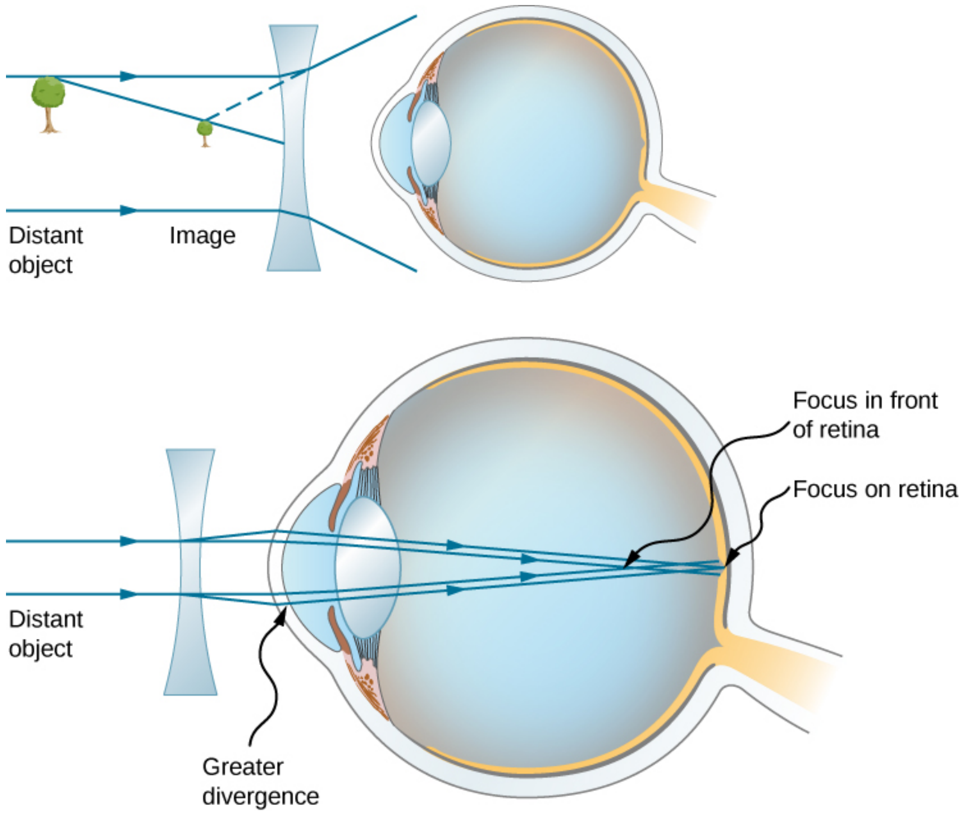


Figure 2.32 Correction of nearsightedness requires a diverging lens that compensates for overconvergence by the eye. The diverging lens produces an image closer to the eye than the physical object. This image serves as the optical object for the eye, and the nearsighted person can see it clearly because it is closer than their far point.

Example 2.8

Correcting Nearsightedness

What optical power of eyeglass lens is needed to correct the vision of a nearsighted person whose far point is 30.0 cm? Assume the corrective lens is fixed 1.50 cm away from the eye.

Strategy

You want this nearsighted person to be able to see distant objects clearly, which means that the eyeglass lens must produce an image 30.0 cm from the eye for an object at infinity. An image 30.0 cm from the eye will be $30.0\,\text{cm} - 1.50\,\text{cm} = 28.5\,\text{cm}$ from the eyeglass lens. Therefore, we must have $d_\text{i} = -28.5\,\text{cm}$ when $d_\text{o} = \infty$. The image distance is negative because it is on the same side of the eyeglass lens as the object.

Solution

Since d_i and d_o are known, we can find the optical power of the eyeglass lens by using **Equation 2.24**:

$$P = \frac{1}{d_\text{o}} + \frac{1}{d_\text{i}} = \frac{1}{\infty} + \frac{1}{-0.285\,\text{m}} = -3.51\text{D}.$$

Significance

The negative optical power indicates a diverging (or concave) lens, as expected. If you examine eyeglasses for nearsighted people, you will find the lenses are thinnest in the center. Additionally, if you examine a prescription for eyeglasses for nearsighted people, you will find that the prescribed optical power is negative and given in units of diopters.

Correcting farsightedness consists simply of using the opposite type of lens as for nearsightedness (i.e., a converging lens), as shown in **Figure 2.33**.

Such a lens will produce an image of physical objects that are closer than the near point at a distance that is between the near point and the far point, so that the person can see the image clearly. To determine the optical power needed for correction, you must therefore know the person's near point, as explained in **Example 2.9**.

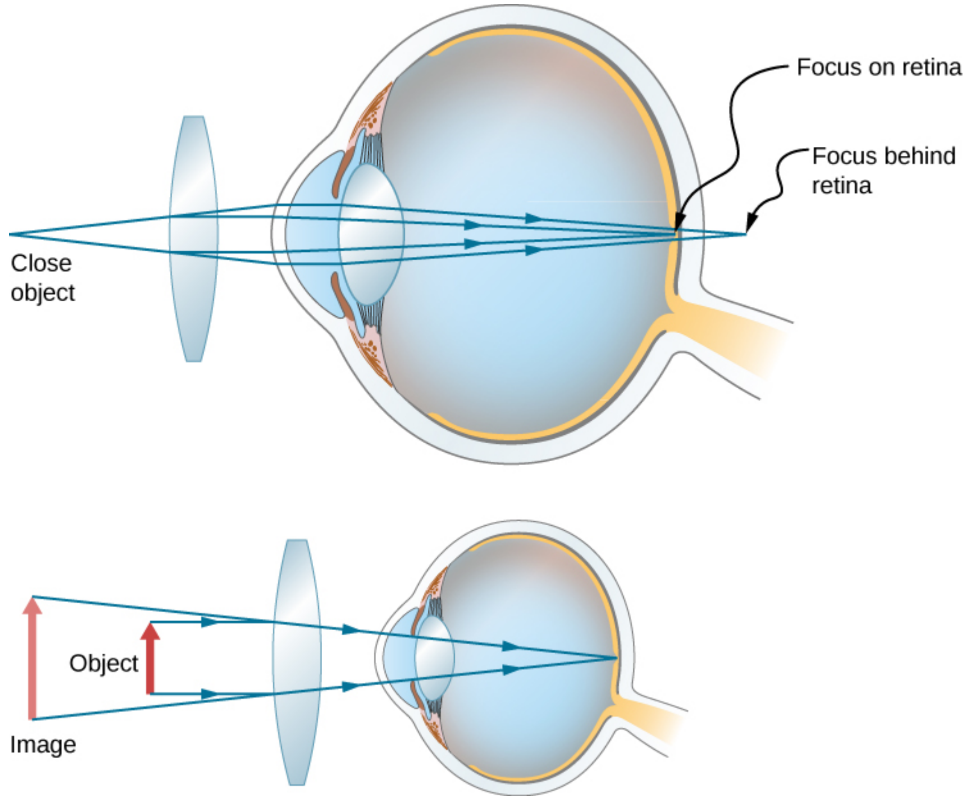


Figure 2.33 Correction of farsightedness uses a converging lens that compensates for the underconvergence by the eye. The converging lens produces an image farther from the eye than the object, so that the farsighted person can see it clearly.

Example 2.9

Correcting Farsightedness

What optical power of eyeglass lens is needed to allow a farsighted person, whose near point is 1.00 m, to see an object clearly that is 25.0 cm from the eye? Assume the corrective lens is fixed 1.5 cm from the eye.

Strategy

When an object is 25.0 cm from the person's eyes, the eyeglass lens must produce an image 1.00 m away (the near point), so that the person can see it clearly. An image 1.00 m from the eye will be $100\,\text{cm} - 1.5\,\text{cm} = 98.5\,\text{cm}$ from the eyeglass lens because the eyeglass lens is 1.5 cm from the eye. Therefore, $d_\text{i} = -98.5\,\text{cm}$, where the minus sign indicates that the image is on the same side of the lens as the object. The object is $25.0\,\text{cm} - 1.5\,\text{cm} = 23.5\,\text{cm}$ from the eyeglass lens, so $d_\text{o} = 23.5\,\text{cm}$.

Solution

Since d_i and d_o are known, we can find the optical power of the eyeglass lens by using **Equation 2.24**:

$$P = \frac{1}{d_\text{o}} + \frac{1}{d_\text{i}} = \frac{1}{0.235\,\text{m}} + \frac{1}{-0.985\,\text{m}} = +3.24\,\text{D}.$$

Significance

The positive optical power indicates a converging (convex) lens, as expected. If you examine eyeglasses of farsighted people, you will find the lenses to be thickest in the center. In addition, prescription eyeglasses for farsighted people have a prescribed optical power that is positive.

2.6 | The Camera

Learning Objectives

By the end of this section, you will be able to:

- Describe the optics of a camera
- Characterize the image created by a camera

Cameras are very common in our everyday life. Between 1825 and 1827, French inventor Nicéphore Niépce successfully photographed an image created by a primitive camera. Since then, enormous progress has been achieved in the design of cameras and camera-based detectors.

Initially, photographs were recorded by using the light-sensitive reaction of silver-based compounds such as silver chloride or silver bromide. Silver-based photographic paper was in common use until the advent of digital photography in the 1980s, which is intimately connected to **charge-coupled device (CCD)** detectors. In a nutshell, a CCD is a semiconductor chip that records images as a matrix of tiny pixels, each pixel located in a "bin" in the surface. Each pixel is capable of detecting the intensity of light impinging on it. Color is brought into play by putting red-, blue-, and green-colored filters over the pixels, resulting in colored digital images (Figure 2.34). At its best resolution, one CCD pixel corresponds to one pixel of the image. To reduce the resolution and decrease the size of the file, we can "bin" several CCD pixels into one, resulting in a smaller but "pixelated" image.

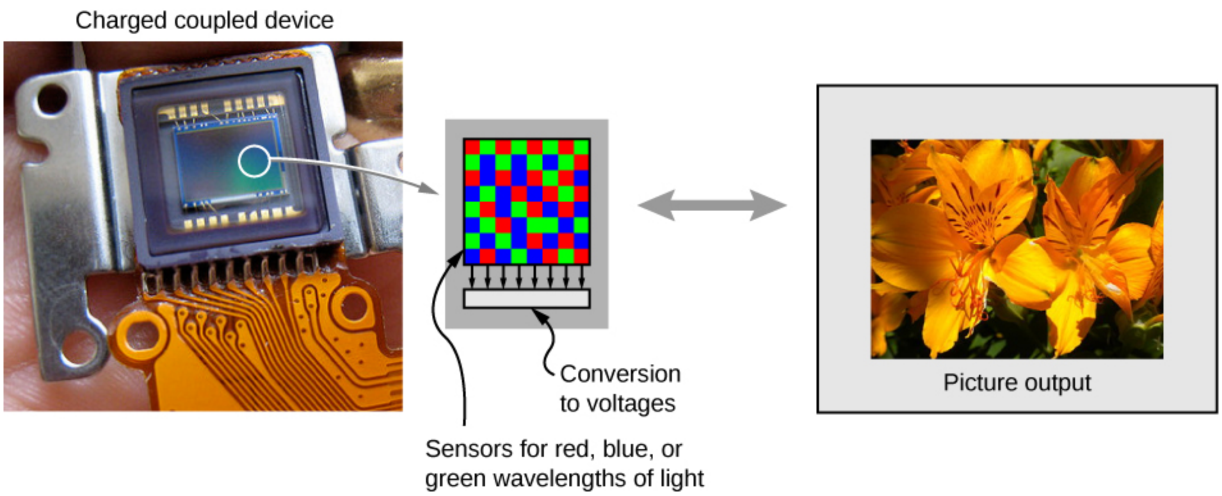


Figure 2.34 A charge-coupled device (CCD) converts light signals into electronic signals, enabling electronic processing and storage of visual images. This is the basis for electronic imaging in all digital cameras, from cell phones to movie cameras. (credit left: modification of work by Bruce Turner)

Clearly, electronics is a big part of a digital camera; however, the underlying physics is basic optics. As a matter of fact, the optics of a camera are pretty much the same as those of a single lens with an object distance that is significantly larger than the lens's focal distance (Figure 2.35).

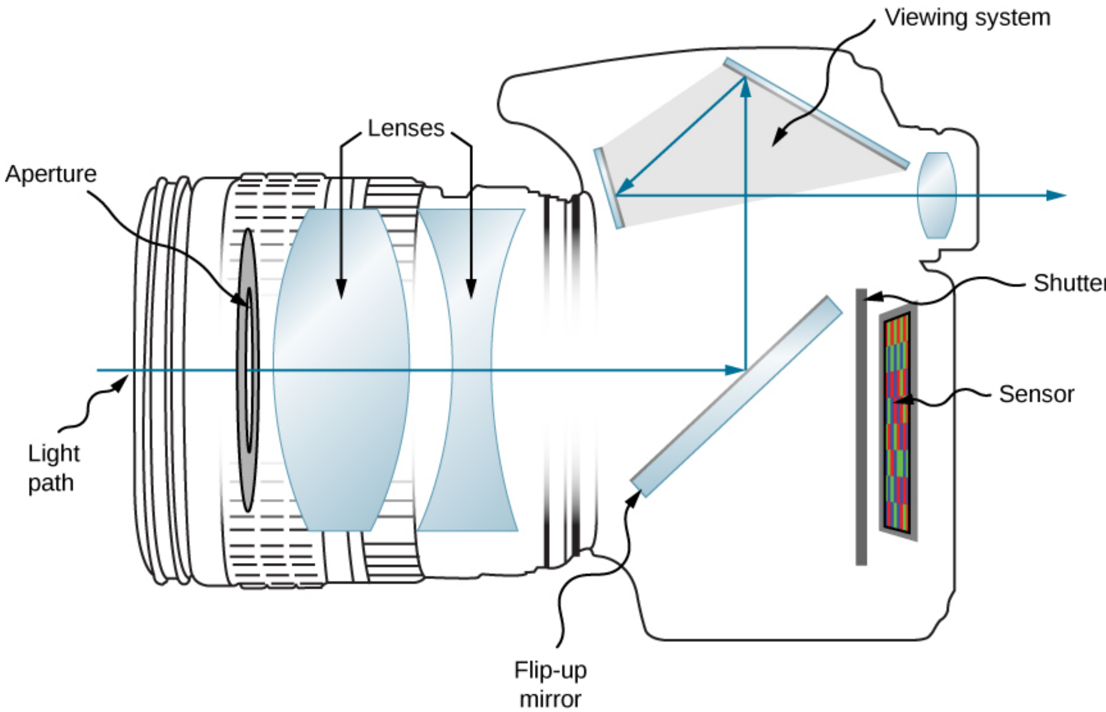


Figure 2.35 Modern digital cameras have several lenses to produce a clear image with minimal aberration and use red, blue, and green filters to produce a color image.

For instance, let us consider the camera in a smartphone. An average smartphone camera is equipped with a stationary wide-angle lens with a focal length of about 4–5 mm. (This focal length is about equal to the thickness of the phone.) The image created by the lens is focused on the CCD detector mounted at the opposite side of the phone. In a cell phone, the lens and the CCD cannot move relative to each other. So how do we make sure that both the images of a distant and a close object are in focus?

Recall that a human eye can accommodate for distant and close images by changing its focal distance. A cell phone camera cannot do that because the distance from the lens to the detector is fixed. Here is where the small focal distance becomes important. Let us assume we have a camera with a 5-mm focal distance. What is the image distance for a selfie? The object distance for a selfie (the length of the hand holding the phone) is about 50 cm. Using the thin-lens equation, we can write

$$\frac{1}{5\text{ mm}} = \frac{1}{500\text{ mm}} + \frac{1}{d_i}$$

We then obtain the image distance:

$$\frac{1}{d_\text{i}} = \frac{1}{5\text{ mm}} - \frac{1}{500\text{ mm}}$$

Note that the object distance is 100 times larger than the focal distance. We can clearly see that the 1/(500 mm) term is significantly smaller than 1/(5 mm), which means that the image distance is pretty much equal to the lens's focal length. An actual calculation gives us the image distance $d_i = 5.05\text{ mm}$. This value is extremely close to the lens's focal distance.

Now let us consider the case of a distant object. Let us say that we would like to take a picture of a person standing about 5 m from us. Using the thin-lens equation again, we obtain the image distance of 5.005 mm. The farther the object is from the lens, the closer the image distance is to the focal distance. At the limiting case of an infinitely distant object, we obtain the image distance exactly equal to the focal distance of the lens.

As you can see, the difference between the image distance for a selfie and the image distance for a distant object is just about 0.05 mm or 50 microns. Even a short object distance such as the length of your hand is two orders of magnitude larger than the lens's focal length, resulting in minute variations of the image distance. (The 50-micron difference is smaller than the thickness of an average sheet of paper.) Such a small difference can be easily accommodated by the same detector, positioned at the focal distance of the lens. Image analysis software can help improve image quality.

Conventional point-and-shoot cameras often use a movable lens to change the lens-to-image distance. Complex lenses of the more expensive mirror reflex cameras allow for superb quality photographic images. The optics of these camera lenses is beyond the scope of this textbook.

2.7 | The Simple Magnifier

Learning Objectives

By the end of this section, you will be able to:

- Understand the optics of a simple magnifier
- Characterize the image created by a simple magnifier

The apparent size of an object perceived by the eye depends on the angle the object subtends from the eye. As shown in **Figure 2.36**, the object at *A* subtends a larger angle from the eye than when it is position at point *B*. Thus, the object at *A* forms a larger image on the retina (see OA') than when it is positioned at *B* (see OB'). Thus, objects that subtend large angles from the eye appear larger because they form larger images on the retina.

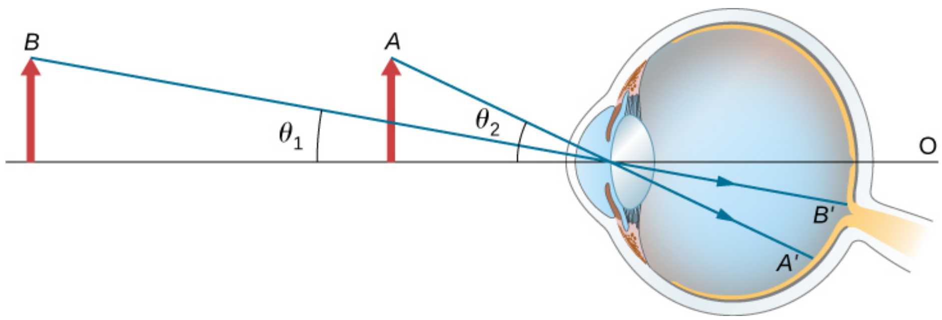


Figure 2.36 Size perceived by an eye is determined by the angle subtended by the object. An image formed on the retina by an object at *A* is larger than an image formed on the retina by the same object positioned at *B* (compared image heights OA' to OB').

We have seen that, when an object is placed within a focal length of a convex lens, its image is virtual, upright, and larger than the object (see part (b) of **Figure 2.26**). Thus, when such an image produced by a convex lens serves as the object for the eye, as shown in **Figure 2.37**, the image on the retina is enlarged, because the image produced by the lens subtends a larger angle in the eye than does the object. A convex lens used for this purpose is called a **magnifying glass** or a **simple magnifier**.

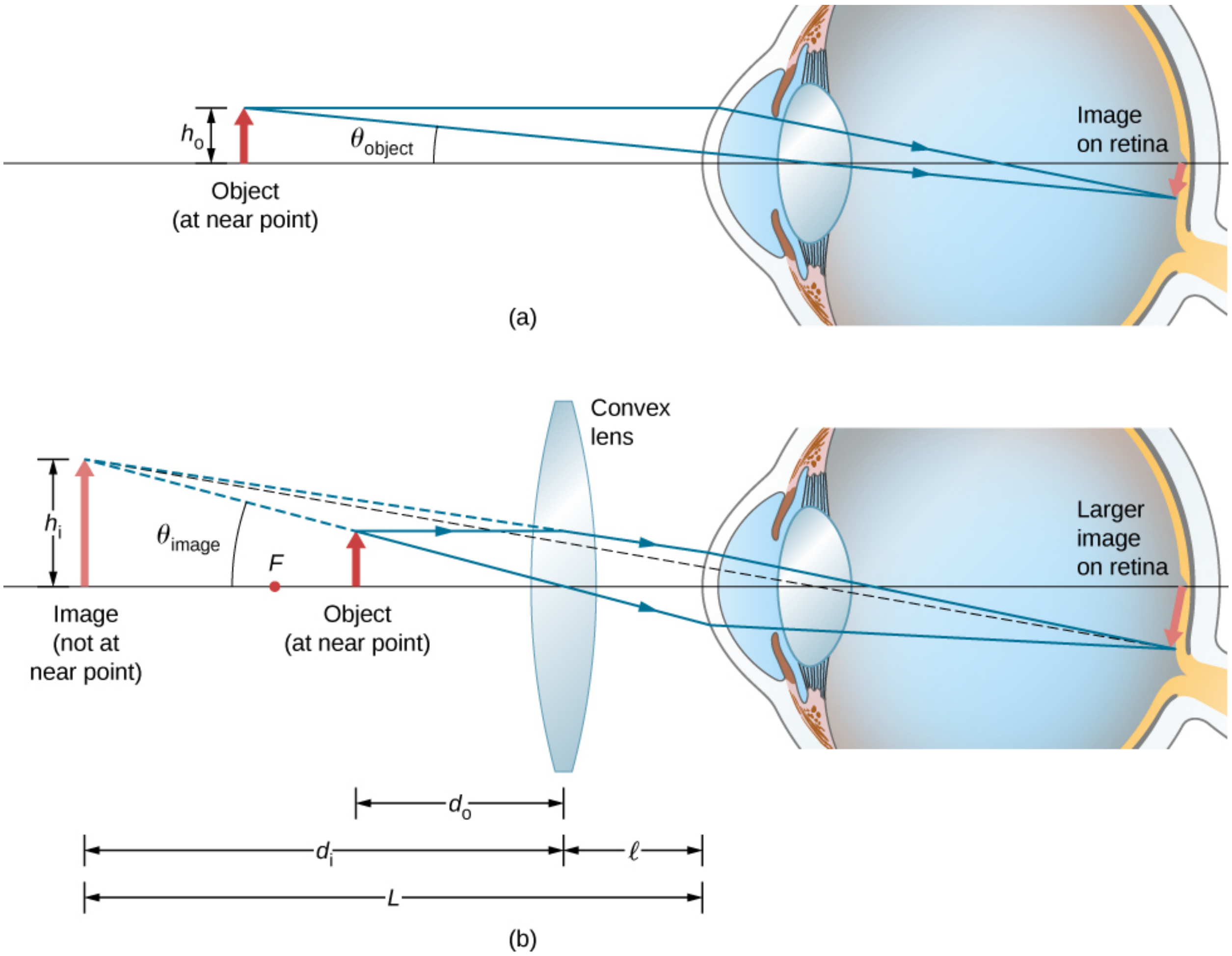


Figure 2.37 The simple magnifier is a convex lens used to produce an enlarged image of an object on the retina. (a) With no convex lens, the object subtends an angle θ_{object} from the eye. (b) With the convex lens in place, the image produced by the convex lens subtends an angle θ_{image} from the eye, with $\theta_{\text{image}} > \theta_{\text{object}}$. Thus, the image on the retina is larger with the convex lens in place.

To account for the magnification of a magnifying lens, we compare the angle subtended by the image (created by the lens) with the angle subtended by the object (viewed with no lens), as shown in **Figure 2.37**. We assume that the object is situated at the near point of the eye, because this is the object distance at which the unaided eye can form the largest image on the retina. We will compare the magnified images created by a lens with this maximum image size for the unaided eye. The magnification of an image when observed by the eye is the **angular magnification** M, which is defined by the ratio of the angle θ_{image} subtended by the image to the angle θ_{object} subtended by the object:

$$M = \frac{\theta_{\text{image}}}{\theta_{\text{object}}}. \tag{2.26}$$

Consider the situation shown in **Figure 2.37**. The magnifying lens is held a distance ℓ from the eye, and the image produced by the magnifier forms a distance L from the eye. We want to calculate the angular magnification for any arbitrary L and ℓ. In the small-angle approximation, the angular size θ_{image} of the image is h_{i}/L. The angular size θ_{object} of the object at the near point is $\theta_{\text{object}} = h_{\text{o}}/25\ \text{cm}$. The angular magnification is then

$$M = \frac{\theta_{\text{image}}}{\theta_{\text{object}}} = \frac{h_{\text{i}}(25\text{ cm})}{Lh_{\text{o}}}. \quad (2.27)$$

Using **Equation 2.8** for linear magnification

$$m = -\frac{d_{\text{i}}}{d_{\text{o}}} = \frac{h_{\text{i}}}{h_{\text{o}}}$$

and the thin-lens equation

$$\frac{1}{d_{\text{o}}} + \frac{1}{d_{\text{i}}} = \frac{1}{f}$$

in **Equation 2.27**, we arrive at the following expression for the angular magnification of a magnifying lens:

$$\begin{aligned} M &= \left(-\frac{d_{\text{i}}}{d_{\text{o}}}\right)\left(\frac{25\text{ cm}}{L}\right) \\ &= -d_{\text{i}}\left(\frac{1}{f} - \frac{1}{d_{\text{i}}}\right)\left(\frac{25\text{ cm}}{L}\right) \\ &= \left(1 - \frac{d_{\text{i}}}{f}\right)\left(\frac{25\text{ cm}}{L}\right) \end{aligned} \quad (2.28)$$

From part (b) of the figure, we see that the absolute value of the image distance is $|d_{\text{i}}| = L - \ell$. Note that $d_{\text{i}} < 0$ because the image is virtual, so we can dispense with the absolute value by explicitly inserting the minus sign: $-d_{\text{i}} = L - \ell$. Inserting this into **Equation 2.28** gives us the final equation for the angular magnification of a magnifying lens:

$$M = \left(\frac{25\text{ cm}}{L}\right)\left(1 + \frac{L - \ell}{f}\right). \quad (2.29)$$

Note that all the quantities in this equation have to be expressed in centimeters. Often, we want the image to be at the near-point distance ($L = 25\text{ cm}$) to get maximum magnification, and we hold the magnifying lens close to the eye ($\ell = 0$). In this case, **Equation 2.29** gives

$$M = 1 + \frac{25\text{ cm}}{f} \quad (2.30)$$

which shows that the greatest magnification occurs for the lens with the shortest focal length. In addition, when the image is at the near-point distance and the lens is held close to the eye $(\ell = 0)$, then $L = d_{\text{i}} = 25\text{ cm}$ and **Equation 2.27** becomes

$$M = \frac{h_{\text{i}}}{h_{\text{o}}} = m \quad (2.31)$$

where *m* is the linear magnification (**Equation 2.32**) derived for spherical mirrors and thin lenses. Another useful situation is when the image is at infinity $(L = \infty)$. **Equation 2.29** then takes the form

$$M(L = \infty) = \frac{25\text{ cm}}{f}. \quad (2.32)$$

The resulting magnification is simply the ratio of the near-point distance to the focal length of the magnifying lens, so a lens with a shorter focal length gives a stronger magnification. Although this magnification is smaller by 1 than the magnification obtained with the image at the near point, it provides for the most comfortable viewing conditions, because the eye is relaxed when viewing a distant object.

By comparing **Equation 2.29** with **Equation 2.32**, we see that the range of angular magnification of a given converging lens is

$$\frac{25\text{ cm}}{f} \le M \le 1 + \frac{25\text{ cm}}{f}. \quad (2.33)$$

Example 2.10

Magnifying a Diamond

A jeweler wishes to inspect a 3.0-mm-diameter diamond with a magnifier. The diamond is held at the jeweler's near point (25 cm), and the jeweler holds the magnifying lens close to his eye.

(a) What should the focal length of the magnifying lens be to see a 15-mm-diameter image of the diamond?

(b) What should the focal length of the magnifying lens be to obtain $10\times$ magnification?

Strategy

We need to determine the requisite magnification of the magnifier. Because the jeweler holds the magnifying lens close to his eye, we can use **Equation 2.30** to find the focal length of the magnifying lens.

Solution

a. The required linear magnification is the ratio of the desired image diameter to the diamond's actual diameter (**Equation 2.32**). Because the jeweler holds the magnifying lens close to his eye and the image forms at his near point, the linear magnification is the same as the angular magnification, so

$$M = m = \frac{h_\text{i}}{h_\text{o}} = \frac{15\text{ mm}}{3.0\text{ mm}} = 5.0.$$

The focal length *f* of the magnifying lens may be calculated by solving **Equation 2.30** for *f*, which gives

$$\begin{aligned} M &= 1 + \frac{25\text{ cm}}{f} \\ f &= \frac{25\text{ cm}}{M - 1} = \frac{25\text{ cm}}{5.0 - 1} = 6.3\text{ cm} \end{aligned}$$

b. To get an image magnified by a factor of ten, we again solve **Equation 2.30** for *f*, but this time we use $M = 10$. The result is

$$f = \frac{25\text{ cm}}{M - 1} = \frac{25\text{ cm}}{10 - 1} = 2.8\text{ cm}.$$

Significance

Note that a greater magnification is achieved by using a lens with a smaller focal length. We thus need to use a lens with radii of curvature that are less than a few centimeters and hold it very close to our eye. This is not very convenient. A compound microscope, explored in the following section, can overcome this drawback.

2.8 | Microscopes and Telescopes

Learning Objectives

By the end of this section, you will be able to:

- Explain the physics behind the operation of microscopes and telescopes
- Describe the image created by these instruments and calculate their magnifications

Microscopes and telescopes are major instruments that have contributed hugely to our current understanding of the micro- and macroscopic worlds. The invention of these devices led to numerous discoveries in disciplines such as physics, astronomy, and biology, to name a few. In this section, we explain the basic physics that make these instruments work.

Microscopes

Although the eye is marvelous in its ability to see objects large and small, it obviously is limited in the smallest details it can detect. The desire to see beyond what is possible with the naked eye led to the use of optical instruments. We have seen that a simple convex lens can create a magnified image, but it is hard to get large magnification with such a lens. A magnification greater than $5\times$ is difficult without distorting the image. To get higher magnification, we can combine the simple magnifying glass with one or more additional lenses. In this section, we examine microscopes that enlarge the details that we cannot see with the naked eye.

Microscopes were first developed in the early 1600s by eyeglass makers in The Netherlands and Denmark. The simplest **compound microscope** is constructed from two convex lenses (Figure 2.38). The **objective** lens is a convex lens of short focal length (i.e., high power) with typical magnification from $5\times$ to $100\times$. The **eyepiece**, also referred to as the ocular, is a convex lens of longer focal length.

The purpose of a microscope is to create magnified images of small objects, and both lenses contribute to the final magnification. Also, the final enlarged image is produced sufficiently far from the observer to be easily viewed, since the eye cannot focus on objects or images that are too close (i.e., closer than the near point of the eye).

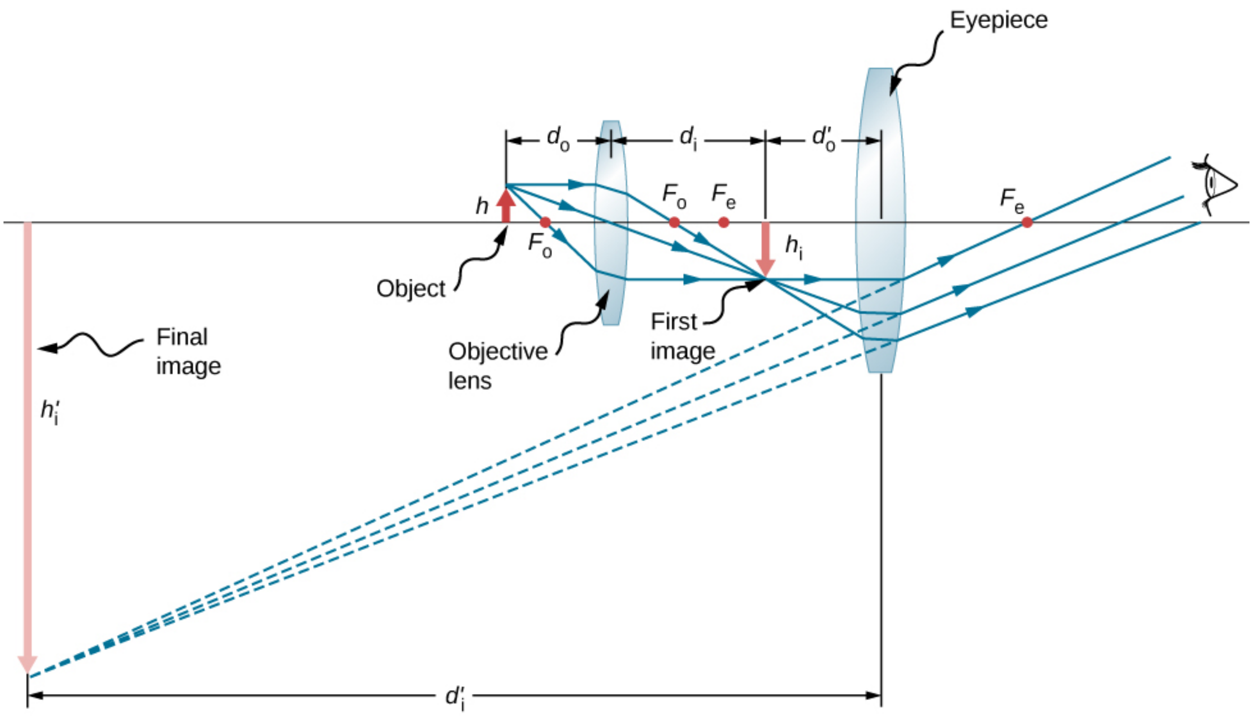


Figure 2.38 A compound microscope is composed of two lenses: an objective and an eyepiece. The objective forms the first image, which is larger than the object. This first image is inside the focal length of the eyepiece and serves as the object for the eyepiece. The eyepiece forms final image that is further magnified.

To see how the microscope in Figure 2.38 forms an image, consider its two lenses in succession. The object is just beyond the focal length f^{obj} of the objective lens, producing a real, inverted image that is larger than the object. This first image serves as the object for the second lens, or eyepiece. The eyepiece is positioned so that the first image is within its focal length f^{eye}, so that it can further magnify the image. In a sense, it acts as a magnifying glass that magnifies the intermediate image produced by the objective. The image produced by the eyepiece is a magnified virtual image. The final image remains inverted but is farther from the observer than the object, making it easy to view.

The eye views the virtual image created by the eyepiece, which serves as the object for the lens in the eye. The virtual image formed by the eyepiece is well outside the focal length of the eye, so the eye forms a real image on the retina.

The magnification of the microscope is the product of the linear magnification m^{obj} by the objective and the angular magnification M^{eye} by the eyepiece. These are given by

$$m^{\text{obj}} = -\frac{d_{\text{i}}^{\text{obj}}}{d_{\text{o}}^{\text{obj}}} \approx -\frac{d_{\text{i}}^{\text{obj}}}{f^{\text{obj}}} \text{(linear magnification y objective)}$$

$$M^{\text{eye}} = 1 + \frac{25\text{ cm}}{f^{\text{eye}}} \text{(angular magnification y eyepiece)}$$

Here, f^{obj} and f^{eye} are the focal lengths of the objective and the eyepiece, respectively. We assume that the final image is formed at the near point of the eye, providing the largest magnification. Note that the angular magnification of the eyepiece is the same as obtained earlier for the simple magnifying glass. This should not be surprising, because the eyepiece is essentially a magnifying glass, and the same physics applies here. The **net magnification** M_{net} of the compound microscope is the product of the linear magnification of the objective and the angular magnification of the eyepiece:

$$M_{\text{net}} = m^{\text{obj}} M^{\text{eye}} = -\frac{d_{\text{i}}^{\text{obj}}\left(f^{\text{eye}} + 25\text{ cm}\right)}{f^{\text{obj}} f^{\text{eye}}}. \tag{2.34}$$

Example 2.11

Microscope Magnification

Calculate the magnification of an object placed 6.20 mm from a compound microscope that has a 6.00 mm-focal length objective and a 50.0 mm-focal length eyepiece. The objective and eyepiece are separated by 23.0 cm.

Strategy

This situation is similar to that shown in **Figure 2.38**. To find the overall magnification, we must know the linear magnification of the objective and the angular magnification of the eyepiece. We can use **Equation 2.34**, but we need to use the thin-lens equation to find the image distance $d_{\text{i}}^{\text{obj}}$ of the objective.

Solution

Solving the thin-lens equation for $d_{\text{i}}^{\text{obj}}$ gives

$$\begin{aligned} d_{\text{i}}^{\text{obj}} &= \left(\frac{1}{f^{\text{obj}}} - \frac{1}{d_{\text{o}}^{\text{obj}}}\right)^{-1} \\ &= \left(\frac{1}{6.00\text{ mm}} - \frac{1}{6.20\text{ mm}}\right)^{-1} = 186\text{ mm} = 18.6\text{ cm} \end{aligned}$$

Inserting this result into **Equation 2.34** along with the known values $f^{\text{obj}} = 6.20\text{ mm} = 0.620\text{ cm}$ and $f^{\text{eye}} = 50.0\text{ mm} = 5.00\text{ cm}$ gives

$$\begin{aligned} M_{\text{net}} &= -\frac{d_{\text{i}}^{\text{obj}}(f^{\text{eye}} + 25\text{ cm})}{f^{\text{obj}} f^{\text{eye}}} \\ &= -\frac{(18.6\text{ cm})(5.00\text{ cm} + 25\text{ cm})}{(0.600\text{ cm})(5.00\text{ cm})} \\ &= -186 \end{aligned}$$

Significance

Both the objective and the eyepiece contribute to the overall magnification, which is large and negative, consistent with **Figure 2.38**, where the image is seen to be large and inverted. In this case, the image is virtual and inverted, which cannot happen for a single element (see **Figure 2.26**).

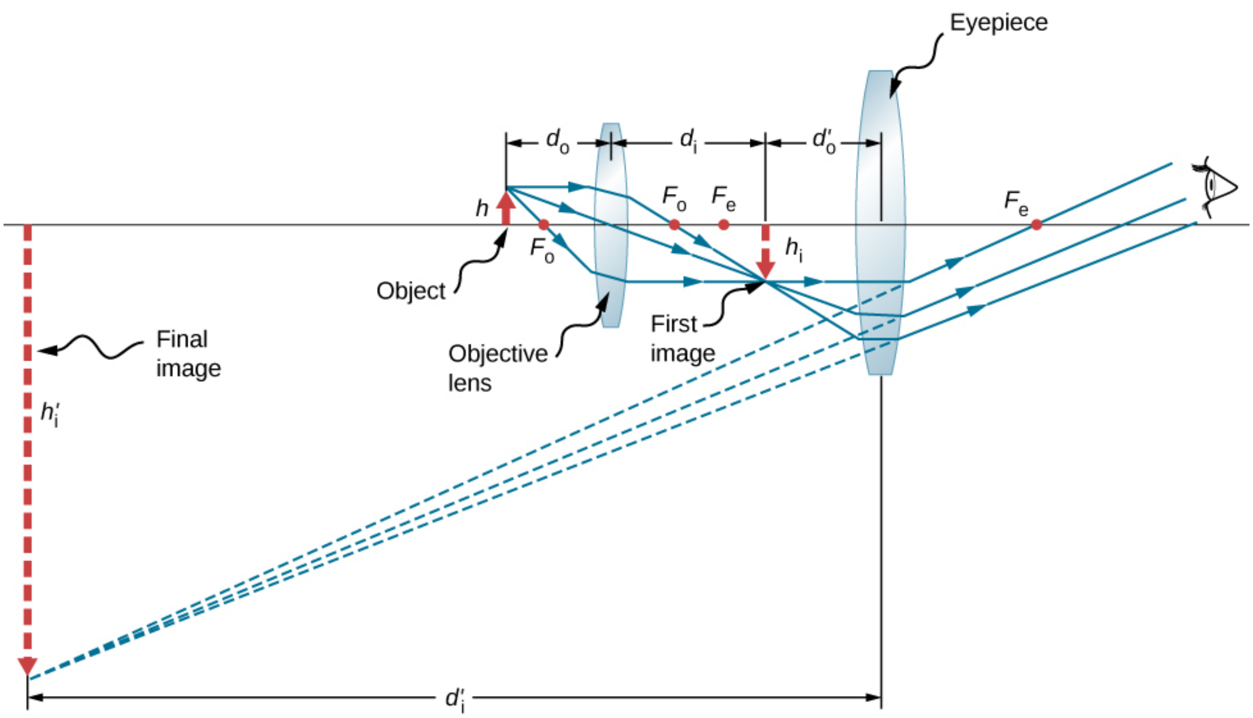


Figure 2.39 A compound microscope with the image created at infinity.

We now calculate the magnifying power of a microscope when the image is at infinity, as shown in **Figure 2.39**, because this makes for the most relaxed viewing. The magnifying power of the microscope is the product of linear magnification m^{obj} of the objective and the angular magnification M^{eye} of the eyepiece. We know that $m^{\text{obj}} = -d_i^{\text{obj}}/d_o^{\text{obj}}$ and from the thin-lens equation we obtain

$$m^{\text{obj}} = -\frac{d_i^{\text{obj}}}{d_o^{\text{obj}}} = 1 - \frac{d_i^{\text{obj}}}{f^{\text{obj}}} = \frac{f^{\text{obj}} - d_i^{\text{obj}}}{f^{\text{obj}}}. \tag{2.35}$$

If the final image is at infinity, then the image created by the objective must be located at the focal point of the eyepiece. This may be seen by considering the thin-lens equation with $d_i = \infty$ or by recalling that rays that pass through the focal point exit the lens parallel to each other, which is equivalent to focusing at infinity. For many microscopes, the distance between the image-side focal point of the objective and the object-side focal point of the eyepiece is standardized at $L = 16\,\text{cm}$. This distance is called the tube length of the microscope. From **Figure 2.39**, we see that $L = f^{\text{obj}} - d_i^{\text{obj}}$. Inserting this into **Equation 2.35** gives

$$m^{\text{obj}} = \frac{L}{f^{\text{obj}}} = \frac{16\,\text{cm}}{f^{\text{obj}}}. \tag{2.36}$$

We now need to calculate the angular magnification of the eyepiece with the image at infinity. To do so, we take the ratio of the angle θ_{image} subtended by the image to the angle θ_{object} subtended by the object at the near point of the eye (this is the closest that the unaided eye can view the object, and thus this is the position where the object will form the largest image on the retina of the unaided eye). Using **Figure 2.39** and working in the small-angle approximation, we have $\theta_{\text{image}} \approx h_i^{\text{obj}}/f^{\text{eye}}$ and $\theta_{\text{object}} \approx h_i^{\text{obj}}/25\,\text{cm}$, where h_i^{obj} is the height of the image formed by the objective, which is the object of the eyepiece. Thus, the angular magnification of the eyepiece is

$$M^{\text{eye}} = \frac{\theta_{\text{image}}}{\theta_{\text{object}}} = \frac{h_i^{\text{obj}}}{f^{\text{eye}}} \frac{25\,\text{cm}}{h_i^{\text{obj}}} = \frac{25\,\text{cm}}{f^{\text{eye}}}. \tag{2.37}$$

The net magnifying power of the compound microscope with the image at infinity is therefore

$$M_{\text{net}} = m^{\text{obj}} M^{\text{eye}} = -\frac{(16\text{ cm})(25\text{ cm})}{f^{\text{obj}} f^{\text{eye}}}. \tag{2.38}$$

The focal distances must be in centimeters. The minus sign indicates that the final image is inverted. Note that the only variables in the equation are the focal distances of the eyepiece and the objective, which makes this equation particularly useful.

Telescopes

Telescopes are meant for viewing distant objects and produce an image that is larger than the image produced in the unaided eye. Telescopes gather far more light than the eye, allowing dim objects to be observed with greater magnification and better resolution. Telescopes were invented around 1600, and Galileo was the first to use them to study the heavens, with monumental consequences. He observed the moons of Jupiter, the craters and mountains on the moon, the details of sunspots, and the fact that the Milky Way is composed of a vast number of individual stars.

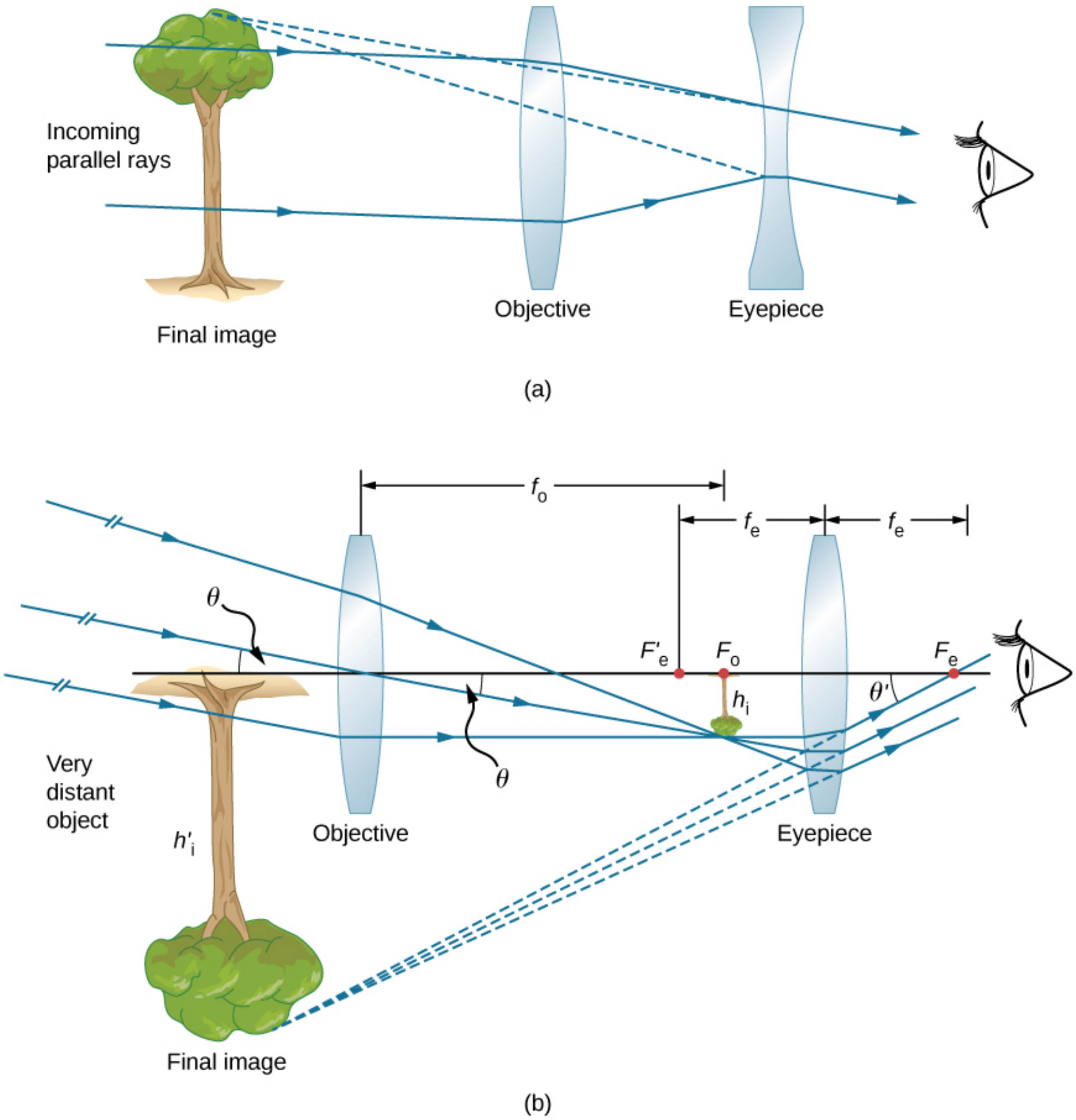


Figure 2.40 (a) Galileo made telescopes with a convex objective and a concave eyepiece. These produce an upright image and are used in spyglasses. (b) Most simple refracting telescopes have two convex lenses. The objective forms a real, inverted image at (or just within) the focal plane of the eyepiece. This image serves as the object for the eyepiece. The eyepiece forms a virtual, inverted image that is magnified.

Part (a) of **Figure 2.40** shows a refracting telescope made of two lenses. The first lens, called the objective, forms a real image within the focal length of the second lens, which is called the eyepiece. The image of the objective lens serves as the object for the eyepiece, which forms a magnified virtual image that is observed by the eye. This design is what Galileo used to observe the heavens.

Although the arrangement of the lenses in a refracting telescope looks similar to that in a microscope, there are important differences. In a telescope, the real object is far away and the intermediate image is smaller than the object. In a microscope, the real object is very close and the intermediate image is larger than the object. In both the telescope and the microscope, the eyepiece magnifies the intermediate image; in the telescope, however, this is the only magnification.

The most common two-lens telescope is shown in part (b) of the figure. The object is so far from the telescope that it is essentially at infinity compared with the focal lengths of the lenses $(d_o^{obj} \approx \infty)$, so the incoming rays are essentially parallel and focus on the focal plane. Thus, the first image is produced at $d_i^{obj} = f^{obj}$, as shown in the figure, and is not large compared with what you might see by looking directly at the object. However, the eyepiece of the telescope eyepiece (like the microscope eyepiece) allows you to get nearer than your near point to this first image and so magnifies it (because you are near to it, it subtends a larger angle from your eye and so forms a larger image on your retina). As for a simple magnifier, the angular magnification of a telescope is the ratio of the angle subtended by the image [θ_{image} in part (b)] to the angle subtended by the real object [θ_{object} in part (b)]:

$$M = \frac{\theta_{image}}{\theta_{object}}. \quad (2.39)$$

To obtain an expression for the magnification that involves only the lens parameters, note that the focal plane of the objective lens lies very close to the focal plan of the eyepiece. If we assume that these planes are superposed, we have the situation shown in Figure 2.41.

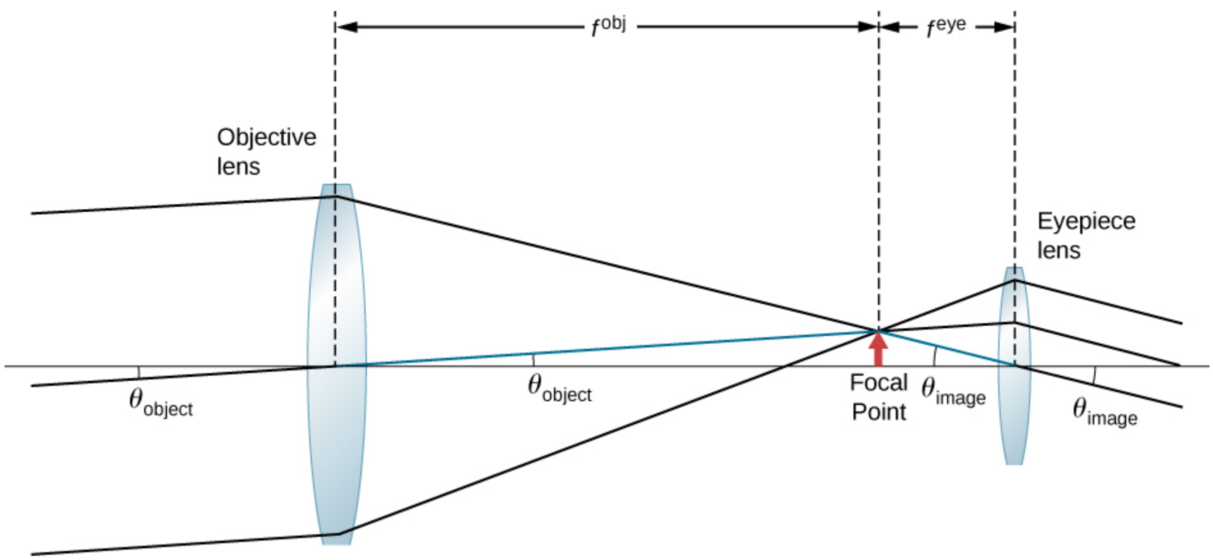


Figure 2.41 The focal plane of the objective lens of a telescope is very near to the focal plane of the eyepiece. The angle θ_{image} subtended by the image viewed through the eyepiece is larger than the angle θ_{object} subtended by the object when viewed with the unaided eye.

We further assume that the angles θ_{object} and θ_{image} are small, so that the small-angle approximation holds ($\tan\theta \approx \theta$). If the image formed at the focal plane has height *h*, then

$$\theta_{object} \approx \tan\theta_{object} = \frac{h}{f^{obj}}$$

$$\theta_{image} \approx \tan\theta_{image} = \frac{-h}{f^{eye}}$$

where the minus sign is introduced because the height is negative if we measure both angles in the counterclockwise direction. Inserting these expressions into **Equation 2.39** gives

$$M = \frac{-h_{\mathrm{i}}}{f^{\mathrm{eye}}} \frac{f^{\mathrm{obj}}}{h_{\mathrm{i}}} = -\frac{f^{\mathrm{obj}}}{f^{\mathrm{eye}}}. \tag{2.40}$$

Thus, to obtain the greatest angular magnification, it is best to have an objective with a long focal length and an eyepiece with a short focal length. The greater the angular magnification *M*, the larger an object will appear when viewed through a telescope, making more details visible. Limits to observable details are imposed by many factors, including lens quality and atmospheric disturbance. Typical eyepieces have focal lengths of 2.5 cm or 1.25 cm. If the objective of the telescope has a focal length of 1 meter, then these eyepieces result in magnifications of $40\times$ and $80\times$, respectively. Thus, the angular magnifications make the image appear 40 times or 80 times closer than the real object.

The minus sign in the magnification indicates the image is inverted, which is unimportant for observing the stars but is a real problem for other applications, such as telescopes on ships or telescopic gun sights. If an upright image is needed, Galileo's arrangement in part (a) of **Figure 2.40** can be used. But a more common arrangement is to use a third convex lens as an eyepiece, increasing the distance between the first two and inverting the image once again, as seen in **Figure 2.42**.

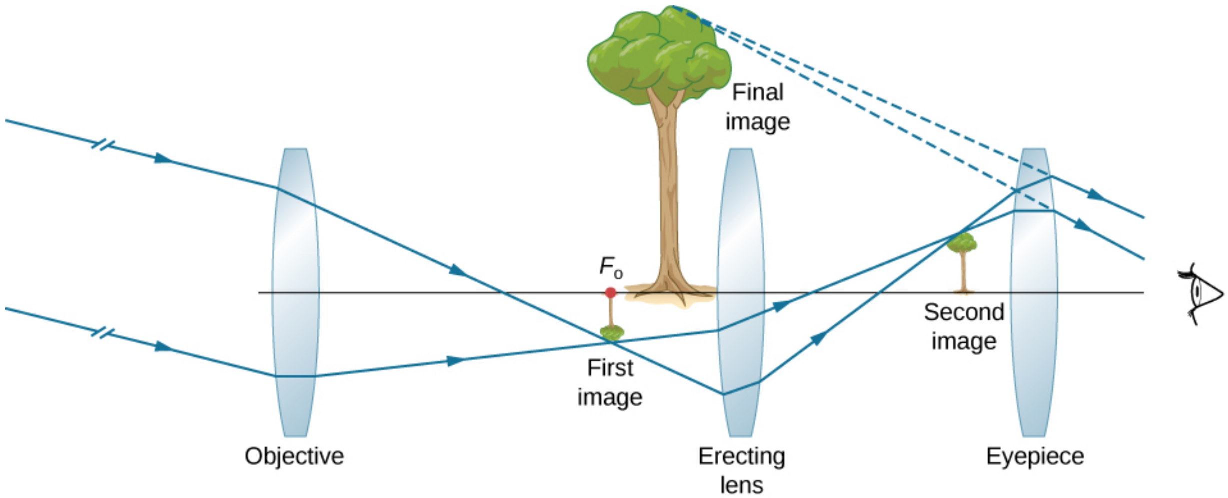


Figure 2.42 This arrangement of three lenses in a telescope produces an upright final image. The first two lenses are far enough apart that the second lens inverts the image of the first. The third lens acts as a magnifier and keeps the image upright and in a location that is easy to view.

The largest refracting telescope in the world is the 40-inch diameter Yerkes telescope located at Lake Geneva, Wisconsin (**Figure 2.43**), and operated by the University of Chicago.

It is very difficult and expensive to build large refracting telescopes. You need large defect-free lenses, which in itself is a technically demanding task. A refracting telescope basically looks like a tube with a support structure to rotate it in different directions. A refracting telescope suffers from several problems. The aberration of lenses causes the image to be blurred. Also, as the lenses become thicker for larger lenses, more light is absorbed, making faint stars more difficult to observe. Large lenses are also very heavy and deform under their own weight. Some of these problems with refracting telescopes are addressed by avoiding refraction for collecting light and instead using a curved mirror in its place, as devised by Isaac Newton. These telescopes are called reflecting telescopes.

Figure 2.43 In 1897, the Yerkes Observatory in Wisconsin (USA) built a large refracting telescope with an objective lens that is 40 inches in diameter and has a tube length of 62 feet. (credit: Yerkes Observatory, University of Chicago)

Reflecting Telescopes

Isaac Newton designed the first reflecting telescope around 1670 to solve the problem of chromatic aberration that happens in all refracting telescopes. In chromatic aberration, light of different colors refracts by slightly different amounts in the lens. As a result, a rainbow appears around the image and the image appears blurred. In the reflecting telescope, light rays from a distant source fall upon the surface of a concave mirror fixed at the bottom end of the tube. The use of a mirror instead of a lens eliminates chromatic aberration. The concave mirror focuses the rays on its focal plane. The design problem is how to observe the focused image. Newton used a design in which the focused light from the concave mirror was reflected to one side of the tube into an eyepiece [part (a) of Figure 2.44]. This arrangement is common in many amateur telescopes and is called the **Newtonian design**.

Some telescopes reflect the light back toward the middle of the concave mirror using a convex mirror. In this arrangement, the light-gathering concave mirror has a hole in the middle [part (b) of the figure]. The light then is incident on an eyepiece lens. This arrangement of the objective and eyepiece is called the **Cassegrain design**. Most big telescopes, including the Hubble space telescope, are of this design. Other arrangements are also possible. In some telescopes, a light detector is placed right at the spot where light is focused by the curved mirror.

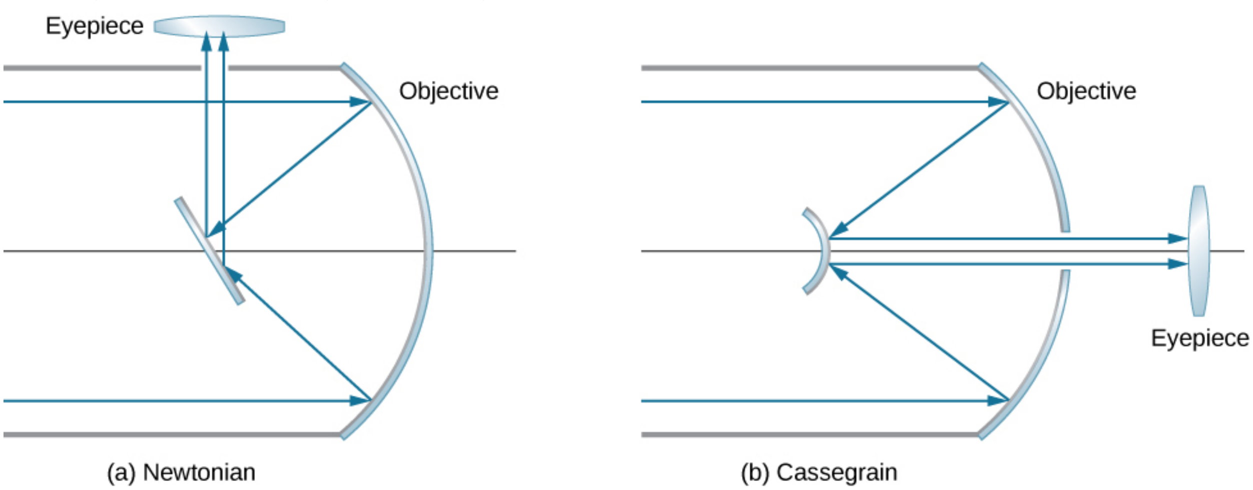


Figure 2.44 Reflecting telescopes: (a) In the Newtonian design, the eyepiece is located at the side of the telescope; (b) in the Cassegrain design, the eyepiece is located past a hole in the primary mirror.

Most astronomical research telescopes are now of the reflecting type. One of the earliest large telescopes of this kind is the Hale 200-inch (or 5-meter) telescope built on Mount Palomar in southern California, which has a 200 inch-diameter mirror. One of the largest telescopes in the world is the 10-meter Keck telescope at the Keck Observatory on the summit of the dormant Mauna Kea volcano in Hawaii. The Keck Observatory operates two 10-meter telescopes. Each is not a single mirror, but is instead made up of 36 hexagonal mirrors. Furthermore, the two telescopes on the Keck can work together, which increases their power to an effective 85-meter mirror. The Hubble telescope (**Figure 2.45**) is another large reflecting telescope with a 2.4 meter-diameter primary mirror. The Hubble was put into orbit around Earth in 1990.

Figure 2.45 The Hubble space telescope as seen from the Space Shuttle Discovery. (credit: modification of work by NASA)

The angular magnification M of a reflecting telescope is also given by **Equation 2.36**. For a spherical mirror, the focal length is half the radius of curvature, so making a large objective mirror not only helps the telescope collect more light but also increases the magnification of the image.

CHAPTER 2 REVIEW

KEY TERMS

aberration distortion in an image caused by departures from the small-angle approximation

accommodation use of the ciliary muscles to adjust the shape of the eye lens for focusing on near or far objects

angular magnification ratio of the angle subtended by an object observed with a magnifier to that observed by the naked eye

apparent depth depth at which an object is perceived to be located with respect to an interface between two media

Cassegrain design arrangement of an objective and eyepiece such that the light-gathering concave mirror has a hole in the middle, and light then is incident on an eyepiece lens

charge-coupled device (CCD) semiconductor chip that converts a light image into tiny pixels that can be converted into electronic signals of color and intensity

coma similar to spherical aberration, but arises when the incoming rays are not parallel to the optical axis

compound microscope microscope constructed from two convex lenses, the first serving as the eyepiece and the second serving as the objective lens

concave mirror spherical mirror with its reflecting surface on the inner side of the sphere; the mirror forms a “cave”

converging (or convex) lens lens in which light rays that enter it parallel converge into a single point on the opposite side

convex mirror spherical mirror with its reflecting surface on the outer side of the sphere

curved mirror mirror formed by a curved surface, such as spherical, elliptical, or parabolic

diverging (or concave) lens lens that causes light rays to bend away from its optical axis

eyepiece lens or combination of lenses in an optical instrument nearest to the eye of the observer

far point furthest point an eye can see in focus

farsightedness (or hyperopia) visual defect in which near objects appear blurred because their images are focused behind the retina rather than on the retina; a farsighted person can see far objects clearly but near objects appear blurred

first focus or object focus object located at this point will result in an image created at infinity on the opposite side of a spherical interface between two media

focal length distance along the optical axis from the focal point to the optical element that focuses the light rays

focal plane plane that contains the focal point and is perpendicular to the optical axis

focal point for a converging lens or mirror, the point at which converging light rays cross; for a diverging lens or mirror, the point from which diverging light rays appear to originate

image distance distance of the image from the central axis of the optical element that produces the image

linear magnification ratio of image height to object height

magnification ratio of image size to object size

near point closest point an eye can see in focus

nearsightedness (or myopia) visual defect in which far objects appear blurred because their images are focused in front of the retina rather than on the retina; a nearsighted person can see near objects clearly but far objects appear blurred

net magnification (M_{net}) of the compound microscope is the product of the linear magnification of the objective and the angular magnification of the eyepiece

Newtonian design arrangement of an objective and eyepiece such that the focused light from the concave mirror was reflected to one side of the tube into an eyepiece

object distance distance of the object from the central axis of the optical element that produces its image

objective lens nearest to the object being examined.

optical axis axis about which the mirror is rotationally symmetric; you can rotate the mirror about this axis without changing anything

optical power (P) inverse of the focal length of a lens, with the focal length expressed in meters. The optical power P of a lens is expressed in units of diopters D; that is, $1\text{D} = 1/\text{m} = 1\ \text{m}^{-1}$

plane mirror plane (flat) reflecting surface

ray tracing technique that uses geometric constructions to find and characterize the image formed by an optical system

real image image that can be projected onto a screen because the rays physically go through the image

second focus or image focus for a converging interface, the point where a bundle of parallel rays refracting at a spherical interface; for a diverging interface, the point at which the backward continuation of the refracted rays will converge between two media will focus

simple magnifier (or magnifying glass) converging lens that produces a virtual image of an object that is within the focal length of the lens

small-angle approximation approximation that is valid when the size of a spherical mirror is significantly smaller than the mirror's radius; in this approximation, spherical aberration is negligible and the mirror has a well-defined focal point

spherical aberration distortion in the image formed by a spherical mirror when rays are not all focused at the same point

thin-lens approximation assumption that the lens is very thin compared to the first image distance

vertex point where the mirror's surface intersects with the optical axis

virtual image image that cannot be projected on a screen because the rays do not physically go through the image, they only appear to originate from the image

KEY EQUATIONS

Image distance in a plane mirror	$d_\text{o} = -d_\text{i}$
Focal length for a spherical mirror	$f = \dfrac{R}{2}$
Mirror equation	$\dfrac{1}{d_\text{o}} + \dfrac{1}{d_\text{i}} = \dfrac{1}{f}$
Magnification of a spherical mirror	$m = \dfrac{h_\text{i}}{h_\text{o}} = -\dfrac{d_\text{i}}{d_\text{o}}$
Sign convention for mirrors	
Focal length f	+ for concave mirror − for convex mirror
Object distance d_o	+ for real object − for virtual object
Image distance d_i	+ for real image − for virtual image
Magnification m	+ for upright image − for inverted image
Apparent depth equation	$h_\text{i} = \left(\dfrac{n_2}{n_1}\right)h_\text{o}$

Spherical interface equation	$\frac{n_1}{d_o} + \frac{n_2}{d_i} = \frac{n_2 - n_1}{R}$
The thin-lens equation	$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$
The lens maker's equation	$\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$
The magnification *m* of an object	$m \equiv \frac{h_i}{h_o} = -\frac{d_i}{d_o}$
Optical power	$P = \frac{1}{f}$
Optical power of thin, closely spaced lenses	$P_{\text{total}} = P_{\text{lens1}} + P_{\text{lens2}} + P_{\text{lens3}} + \cdots$
Angular magnification *M* of a simple magnifier	$M = \frac{\theta_{\text{image}}}{\theta_{\text{object}}}$
Angular magnification of an object a distance *L* from the eye for a convex lens of focal length *f* held a distance *ℓ* from the eye	$M = \left(\frac{25\text{ cm}}{L}\right)\left(1 + \frac{L - \ell}{f}\right)$
Range of angular magnification for a given lens for a person with a near point of 25 cm	$\frac{25\text{ cm}}{f} \le M \le 1 + \frac{25\text{ cm}}{f}$
Net magnification of compound microscope	$M_{\text{net}} = m^{\text{obj}} M^{\text{eye}} = -\frac{d_i^{\text{obj}}\left(f^{\text{eye}} + 25\text{ cm}\right)}{f^{\text{obj}} f^{\text{eye}}}$

SUMMARY

2.1 Images Formed by Plane Mirrors

- A plane mirror always forms a virtual image (behind the mirror).
- The image and object are the same distance from a flat mirror, the image size is the same as the object size, and the image is upright.

2.2 Spherical Mirrors

- Spherical mirrors may be concave (converging) or convex (diverging).
- The focal length of a spherical mirror is one-half of its radius of curvature: $f = R/2$.
- The mirror equation and ray tracing allow you to give a complete description of an image formed by a spherical mirror.
- Spherical aberration occurs for spherical mirrors but not parabolic mirrors; comatic aberration occurs for both types of mirrors.

2.3 Images Formed by Refraction

This section explains how a single refracting interface forms images.

- When an object is observed through a plane interface between two media, then it appears at an apparent distance h_i that differs from the actual distance h_o: $h_i = (n_2/n_1)h_o$.
- An image is formed by the refraction of light at a spherical interface between two media of indices of refraction n_1 and n_2.

- Image distance depends on the radius of curvature of the interface, location of the object, and the indices of refraction of the media.

2.4 Thin Lenses

- Two types of lenses are possible: converging and diverging. A lens that causes light rays to bend toward (away from) its optical axis is a converging (diverging) lens.
- For a converging lens, the focal point is where the converging light rays cross; for a diverging lens, the focal point is the point from which the diverging light rays appear to originate.
- The distance from the center of a thin lens to its focal point is called the focal length *f*.
- Ray tracing is a geometric technique to determine the paths taken by light rays through thin lenses.
- A real image can be projected onto a screen.
- A virtual image cannot be projected onto a screen.
- A converging lens forms either real or virtual images, depending on the object location; a diverging lens forms only virtual images.

2.5 The Eye

- Image formation by the eye is adequately described by the thin-lens equation.
- The eye produces a real image on the retina by adjusting its focal length in a process called accommodation.
- Nearsightedness, or myopia, is the inability to see far objects and is corrected with a diverging lens to reduce the optical power of the eye.
- Farsightedness, or hyperopia, is the inability to see near objects and is corrected with a converging lens to increase the optical power of the eye.
- In myopia and hyperopia, the corrective lenses produce images at distances that fall between the person's near and far points so that images can be seen clearly.

2.6 The Camera

- Cameras use combinations of lenses to create an image for recording.
- Digital photography is based on charge-coupled devices (CCDs) that break an image into tiny "pixels" that can be converted into electronic signals.

2.7 The Simple Magnifier

- A simple magnifier is a converging lens and produces a magnified virtual image of an object located within the focal length of the lens.
- Angular magnification accounts for magnification of an image created by a magnifier. It is equal to the ratio of the angle subtended by the image to that subtended by the object when the object is observed by the unaided eye.
- Angular magnification is greater for magnifying lenses with smaller focal lengths.
- Simple magnifiers can produce as great as tenfold ($10\times$) magnification.

2.8 Microscopes and Telescopes

- Many optical devices contain more than a single lens or mirror. These are analyzed by considering each element sequentially. The image formed by the first is the object for the second, and so on. The same ray-tracing and thin-lens techniques developed in the previous sections apply to each lens element.
- The overall magnification of a multiple-element system is the product of the linear magnifications of its individual elements times the angular magnification of the eyepiece. For a two-element system with an objective and an eyepiece, this is

$$M = m^{\text{obj}} M^{\text{eye}}.$$

where m^{obj} is the linear magnification of the objective and M^{eye} is the angular magnification of the eyepiece.

- The microscope is a multiple-element system that contains more than a single lens or mirror. It allows us to see detail that we could not to see with the unaided eye. Both the eyepiece and objective contribute to the magnification. The magnification of a compound microscope with the image at infinity is

$$M_{\text{net}} = -\frac{(16\text{ cm})(25\text{ cm})}{f^{\text{obj}} f^{\text{eye}}}.$$

In this equation, 16 cm is the standardized distance between the image-side focal point of the objective lens and the object-side focal point of the eyepiece, 25 cm is the normal near point distance, f^{obj} and f^{eye} are the focal distances for the objective lens and the eyepiece, respectively.

- Simple telescopes can be made with two lenses. They are used for viewing objects at large distances.
- The angular magnification *M* for a telescope is given by

$$M = -\frac{f^{\text{obj}}}{f^{\text{eye}}},$$

where f^{obj} and f^{eye} are the focal lengths of the objective lens and the eyepiece, respectively.

CONCEPTUAL QUESTIONS

2.1 Images Formed by Plane Mirrors

1. What are the differences between real and virtual images? How can you tell (by looking) whether an image formed by a single lens or mirror is real or virtual?

2. Can you see a virtual image? Explain your response.

3. Can you photograph a virtual image?

4. Can you project a virtual image onto a screen?

5. Is it necessary to project a real image onto a screen to see it?

6. Devise an arrangement of mirrors allowing you to see the back of your head. What is the minimum number of mirrors needed for this task?

7. If you wish to see your entire body in a flat mirror (from head to toe), how tall should the mirror be? Does its size depend upon your distance away from the mirror? Provide a sketch.

2.2 Spherical Mirrors

8. At what distance is an image always located: at d_o, d_i, or f?

9. Under what circumstances will an image be located at the focal point of a spherical lens or mirror?

10. What is meant by a negative magnification? What is meant by a magnification whose absolute value is less than one?

11. Can an image be larger than the object even though its magnification is negative? Explain.

2.3 Images Formed by Refraction

12. Derive the formula for the apparent depth of a fish in a fish tank using Snell's law.

13. Use a ruler and a protractor to find the image by refraction in the following cases. Assume an air-glass interface. Use a refractive index of 1 for air and of 1.5 for glass. (*Hint*: Use Snell's law at the interface.)

(a) A point object located on the axis of a concave interface located at a point within the focal length from the vertex.

(b) A point object located on the axis of a concave interface located at a point farther than the focal length from the vertex.

(c) A point object located on the axis of a convex interface located at a point within the focal length from the vertex.

(d) A point object located on the axis of a convex interface located at a point farther than the focal length from the vertex.

(e) Repeat (a)–(d) for a point object off the axis.

2.4 Thin Lenses

14. You can argue that a flat piece of glass, such as in a window, is like a lens with an infinite focal length. If so, where does it form an image? That is, how are d_i and d_o related?

15. When you focus a camera, you adjust the distance of the lens from the film. If the camera lens acts like a thin lens, why can it not be a fixed distance from the film for both near and distant objects?

16. A thin lens has two focal points, one on either side of the lens at equal distances from its center, and should behave the same for light entering from either side. Look backward and forward through a pair of eyeglasses and comment on whether they are thin lenses.

17. Will the focal length of a lens change when it is submerged in water? Explain.

2.5 The Eye

18. If the lens of a person's eye is removed because of cataracts (as has been done since ancient times), why would you expect an eyeglass lens of about 16 D to be prescribed?

19. When laser light is shone into a relaxed normal-vision eye to repair a tear by spot-welding the retina to the back of the eye, the rays entering the eye must be parallel. Why?

20. Why is your vision so blurry when you open your eyes while swimming under water? How does a face mask enable clear vision?

21. It has become common to replace the cataract-clouded lens of the eye with an internal lens. This intraocular lens can be chosen so that the person has perfect distant vision. Will the person be able to read without glasses? If the person was nearsighted, is the power of the intraocular lens greater or less than the removed lens?

22. If the cornea is to be reshaped (this can be done surgically or with contact lenses) to correct myopia, should its curvature be made greater or smaller? Explain.

2.8 Microscopes and Telescopes

23. Geometric optics describes the interaction of light with macroscopic objects. Why, then, is it correct to use geometric optics to analyze a microscope's image?

24. The image produced by the microscope in **Figure 2.38** cannot be projected. Could extra lenses or mirrors project it? Explain.

25. If you want your microscope or telescope to project a real image onto a screen, how would you change the placement of the eyepiece relative to the objective?

PROBLEMS

2.1 Images Formed by Plane Mirrors

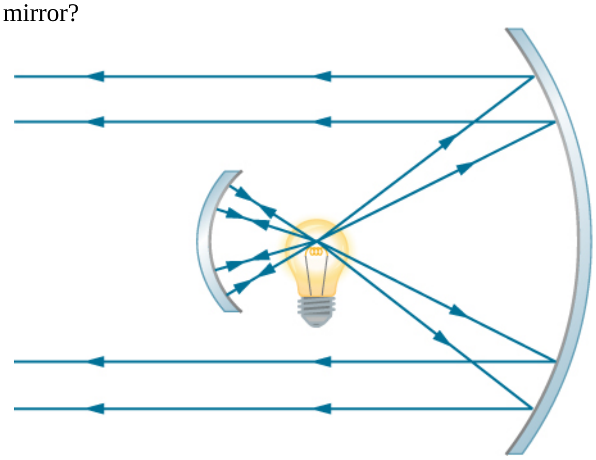
26. Consider a pair of flat mirrors that are positioned so that they form an angle of $120°$. An object is placed on the bisector between the mirrors. Construct a ray diagram as in **Figure 2.4** to show how many images are formed.

27. Consider a pair of flat mirrors that are positioned so that they form an angle of $60°$. An object is placed on the bisector between the mirrors. Construct a ray diagram as in **Figure 2.4** to show how many images are formed.

28. By using more than one flat mirror, construct a ray diagram showing how to create an inverted image.

2.2 Spherical Mirrors

29. The following figure shows a light bulb between two spherical mirrors. One mirror produces a beam of light with parallel rays; the other keeps light from escaping without being put into the beam. Where is the filament of the light in relation to the focal point or radius of curvature of each mirror?



30. Why are diverging mirrors often used for rearview mirrors in vehicles? What is the main disadvantage of using such a mirror compared with a flat one?

31. Some telephoto cameras use a mirror rather than a lens. What radius of curvature mirror is needed to replace a 800 mm-focal length telephoto lens?

32. Calculate the focal length of a mirror formed by the shiny back of a spoon that has a 3.00 cm radius of curvature.

33. Electric room heaters use a concave mirror to reflect infrared (IR) radiation from hot coils. Note that IR radiation follows the same law of reflection as visible light. Given that the mirror has a radius of curvature of 50.0 cm and produces an image of the coils 3.00 m away from the mirror, where are the coils?

34. Find the magnification of the heater element in the previous problem. Note that its large magnitude helps spread out the reflected energy.

35. What is the focal length of a makeup mirror that produces a magnification of 1.50 when a person's face is 12.0 cm away? Explicitly show how you follow the steps in the **Problem-Solving Strategy: Spherical Mirrors**.

36. A shopper standing 3.00 m from a convex security mirror sees his image with a magnification of 0.250. (a) Where is his image? (b) What is the focal length of the mirror? (c) What is its radius of curvature?

37. An object 1.50 cm high is held 3.00 cm from a person's cornea, and its reflected image is measured to be 0.167 cm high. (a) What is the magnification? (b) Where is the image? (c) Find the radius of curvature of the convex mirror formed by the cornea. (Note that this technique is used by optometrists to measure the curvature of the cornea for contact lens fitting. The instrument used is called a keratometer, or curve measurer.)

38. Ray tracing for a flat mirror shows that the image is located a distance behind the mirror equal to the distance of the object from the mirror. This is stated as $d_\text{i} = -d_\text{o}$, since this is a negative image distance (it is a virtual image). What is the focal length of a flat mirror?

39. Show that, for a flat mirror, $h_\text{i} = h_\text{o}$, given that the image is the same distance behind the mirror as the distance of the object from the mirror.

40. Use the law of reflection to prove that the focal length of a mirror is half its radius of curvature. That is, prove that $f = R/2$. Note this is true for a spherical mirror only if its diameter is small compared with its radius of curvature.

41. Referring to the electric room heater considered in problem 5, calculate the intensity of IR radiation in W/m^2 projected by the concave mirror on a person 3.00 m away. Assume that the heating element radiates 1500 W and has an area of $100\ \text{cm}^2$, and that half of the radiated power is reflected and focused by the mirror.

42. Two mirrors are inclined at an angle of $60°$ and an object is placed at a point that is equidistant from the two mirrors. Use a protractor to draw rays accurately and locate all images. You may have to draw several figures so that that rays for different images do not clutter your drawing.

43. Two parallel mirrors are facing each other and are separated by a distance of 3 cm. A point object is placed between the mirrors 1 cm from one of the mirrors. Find the coordinates of all the images.

2.3 Images Formed by Refraction

44. An object is located in air 30 cm from the vertex of a concave surface made of glass with a radius of curvature 10 cm. Where does the image by refraction form and what is its magnification? Use $n_\text{air} = 1$ and $n_\text{glass} = 1.5$.

45. An object is located in air 30 cm from the vertex of a convex surface made of glass with a radius of curvature 80 cm. Where does the image by refraction form and what is its magnification?

46. An object is located in water 15 cm from the vertex of a concave surface made of glass with a radius of curvature 10 cm. Where does the image by refraction form and what is its magnification? Use $n_\text{water} = 4/3$ and $n_\text{glass} = 1.5$.

47. An object is located in water 30 cm from the vertex of a convex surface made of Plexiglas with a radius of curvature of 80 cm. Where does the image form by refraction and what is its magnification? $n_\text{water} = 4/3$ and $n_\text{Plexiglas} = 1.65$.

48. An object is located in air 5 cm from the vertex of a concave surface made of glass with a radius of curvature 20 cm. Where does the image form by refraction and what is its magnification? Use $n_\text{air} = 1$ and $n_\text{glass} = 1.5$.

49. Derive the spherical interface equation for refraction at a concave surface. (*Hint*: Follow the derivation in the text for the convex surface.)

2.4 Thin Lenses

50. How far from the lens must the film in a camera be, if the lens has a 35.0-mm focal length and is being used to photograph a flower 75.0 cm away? Explicitly show how you follow the steps in the **Problem-Solving Strategy: Lenses**.

51. A certain slide projector has a 100 mm-focal length lens. (a) How far away is the screen if a slide is placed 103 mm from the lens and produces a sharp image? (b) If the slide is 24.0 by 36.0 mm, what are the dimensions of the image? Explicitly show how you follow the steps in the **Problem-Solving Strategy: Lenses**.

52. A doctor examines a mole with a 15.0-cm focal length magnifying glass held 13.5 cm from the mole. (a) Where is the image? (b) What is its magnification? (c) How big is the image of a 5.00 mm diameter mole?

53. A camera with a 50.0-mm focal length lens is being used to photograph a person standing 3.00 m away. (a) How far from the lens must the film be? (b) If the film is 36.0 mm high, what fraction of a 1.75-m-tall person will fit on it? (c) Discuss how reasonable this seems, based on your experience in taking or posing for photographs.

54. A camera lens used for taking close-up photographs has a focal length of 22.0 mm. The farthest it can be placed from the film is 33.0 mm. (a) What is the closest object that can be photographed? (b) What is the magnification of this closest object?

55. Suppose your 50.0 mm-focal length camera lens is 51.0 mm away from the film in the camera. (a) How far away is an object that is in focus? (b) What is the height of the object if its image is 2.00 cm high?

56. What is the focal length of a magnifying glass that produces a magnification of 3.00 when held 5.00 cm from an object, such as a rare coin?

57. The magnification of a book held 7.50 cm from a 10.0 cm-focal length lens is 3.00. (a) Find the magnification for the book when it is held 8.50 cm from the magnifier. (b) Repeat for the book held 9.50 cm from the magnifier. (c) Comment on how magnification changes as the object distance increases as in these two calculations.

58. Suppose a 200 mm-focal length telephoto lens is being used to photograph mountains 10.0 km away. (a) Where is the image? (b) What is the height of the image of a 1000 m high cliff on one of the mountains?

59. A camera with a 100 mm-focal length lens is used to photograph the sun. What is the height of the image of the sun on the film, given the sun is 1.40×10^6 km in diameter and is 1.50×10^8 km away?

60. Use the thin-lens equation to show that the magnification for a thin lens is determined by its focal length and the object distance and is given by $m = f/(f - d_o)$.

61. An object of height 3.0 cm is placed 5.0 cm in front of a converging lens of focal length 20 cm and observed from the other side. Where and how large is the image?

62. An object of height 3.0 cm is placed at 5.0 cm in front of a diverging lens of focal length 20 cm and observed from the other side. Where and how large is the image?

63. An object of height 3.0 cm is placed at 25 cm in front of a diverging lens of focal length 20 cm. Behind the diverging lens, there is a converging lens of focal length 20 cm. The distance between the lenses is 5.0 cm. Find the location and size of the final image.

64. Two convex lenses of focal lengths 20 cm and 10 cm are placed 30 cm apart, with the lens with the longer focal length on the right. An object of height 2.0 cm is placed midway between them and observed through each lens from the left and from the right. Describe what you will see, such as where the image(s) will appear, whether they will be upright or inverted and their magnifications.

2.5 The Eye

Unless otherwise stated, the lens-to-retina distance is 2.00 cm.

65. What is the power of the eye when viewing an object 50.0 cm away?

66. Calculate the power of the eye when viewing an object 3.00 m away.

67. The print in many books averages 3.50 mm in height. How high is the image of the print on the retina when the book is held 30.0 cm from the eye?

68. Suppose a certain person's visual acuity is such that he can see objects clearly that form an image $4.00\,\mu\text{m}$ high on his retina. What is the maximum distance at which he can read the 75.0-cm-high letters on the side of an airplane?

69. People who do very detailed work close up, such as jewelers, often can see objects clearly at much closer distance than the normal 25 cm. (a) What is the power of the eyes of a woman who can see an object clearly at a distance of only 8.00 cm? (b) What is the image size of a 1.00-mm object, such as lettering inside a ring, held at this distance? (c) What would the size of the image be if the object were held at the normal 25.0 cm distance?

70. What is the far point of a person whose eyes have a relaxed power of 50.5 D?

71. What is the near point of a person whose eyes have an accommodated power of 53.5 D?

72. (a) A laser reshaping the cornea of a myopic patient reduces the power of his eye by 9.00 D, with a $\pm 5.0\ \%$ uncertainty in the final correction. What is the range of diopters for eyeglass lenses that this person might need after this procedure? (b) Was the person nearsighted or farsighted before the procedure? How do you know?

73. The power for normal close vision is 54.0 D. In a vision-correction procedure, the power of a patient's eye is increased by 3.00 D. Assuming that this produces normal close vision, what was the patient's near point before the procedure?

74. For normal distant vision, the eye has a power of 50.0 D. What was the previous far point of a patient who had laser vision correction that reduced the power of her eye by 7.00 D, producing normal distant vision?

75. The power for normal distant vision is 50.0 D. A severely myopic patient has a far point of 5.00 cm. By how many diopters should the power of his eye be reduced in laser vision correction to obtain normal distant vision for him?

76. A student's eyes, while reading the blackboard, have a power of 51.0 D. How far is the board from his eyes?

77. The power of a physician's eyes is 53.0 D while examining a patient. How far from her eyes is the object that is being examined?

78. The normal power for distant vision is 50.0 D. A young woman with normal distant vision has a 10.0% ability to accommodate (that is, increase) the power of her eyes. What is the closest object she can see clearly?

79. The far point of a myopic administrator is 50.0 cm. (a) What is the relaxed power of his eyes? (b) If he has the normal 8.00% ability to accommodate, what is the closest object he can see clearly?

80. A very myopic man has a far point of 20.0 cm. What power contact lens (when on the eye) will correct his distant vision?

81. Repeat the previous problem for eyeglasses held 1.50 cm from the eyes.

82. A myopic person sees that her contact lens prescription is –4.00 D. What is her far point?

83. Repeat the previous problem for glasses that are 1.75 cm from the eyes.

84. The contact lens prescription for a mildly farsighted person is 0.750 D, and the person has a near point of 29.0 cm. What is the power of the tear layer between the cornea and the lens if the correction is ideal, taking the tear layer into account?

2.7 The Simple Magnifier

85. If the image formed on the retina subtends an angle of $30°$ and the object subtends an angle of $5°$, what is the magnification of the image?

86. What is the magnification of a magnifying lens with a focal length of 10 cm if it is held 3.0 cm from the eye and the object is 12 cm from the eye?

87. How far should you hold a 2.1 cm-focal length magnifying glass from an object to obtain a magnification of $10\times$? Assume you place your eye 5.0 cm from the magnifying glass.

88. You hold a 5.0 cm-focal length magnifying glass as close as possible to your eye. If you have a normal near point, what is the magnification?

89. You view a mountain with a magnifying glass of focal length $f = 10\ \text{cm}$. What is the magnification?

90. You view an object by holding a 2.5 cm-focal length magnifying glass 10 cm away from it. How far from your eye should you hold the magnifying glass to obtain a magnification of $10\times$?

91. A magnifying glass forms an image 10 cm on the opposite side of the lens from the object, which is 10 cm away. What is the magnification of this lens for a person with a normal near point if their eye 12 cm from the object?

92. An object viewed with the naked eye subtends a $2°$ angle. If you view the object through a $10\times$ magnifying glass, what angle is subtended by the image formed on your retina?

93. For a normal, relaxed eye, a magnifying glass produces an angular magnification of 4.0. What is the largest magnification possible with this magnifying glass?

94. What range of magnification is possible with a 7.0 cm-focal length converging lens?

95. A magnifying glass produces an angular magnification of 4.5 when used by a young person with a near point of 18 cm. What is the maximum angular magnification obtained by an older person with a near point of 45 cm?

2.8 Microscopes and Telescopes

96. A microscope with an overall magnification of 800 has an objective that magnifies by 200. (a) What is the angular magnification of the eyepiece? (b) If there are two other objectives that can be used, having magnifications of 100 and 400, what other total magnifications are possible?

97. (a) What magnification is produced by a 0.150 cm-focal length microscope objective that is 0.155 cm from the object being viewed? (b) What is the overall magnification if an $8\times$ eyepiece (one that produces an angular magnification of 8.00) is used?

98. Where does an object need to be placed relative to a microscope for its 0.50 cm-focal length objective to produce a magnification of -400?

99. An amoeba is 0.305 cm away from the 0.300 cm-focal length objective lens of a microscope. (a) Where is the image formed by the objective lens? (b) What is this image's magnification? (c) An eyepiece with a 2.00-cm focal length is placed 20.0 cm from the objective. Where is the final image? (d) What angular magnification is produced by the eyepiece? (e) What is the overall magnification? (See **Figure 2.39**.)

100. **Unreasonable Results** Your friends show you an image through a microscope. They tell you that the microscope has an objective with a 0.500-cm focal length and an eyepiece with a 5.00-cm focal length. The resulting overall magnification is 250,000. Are these viable values for a microscope?

Unless otherwise stated, the lens-to-retina distance is 2.00 cm.

101. What is the angular magnification of a telescope that has a 100 cm-focal length objective and a 2.50 cm-focal length eyepiece?

102. Find the distance between the objective and eyepiece lenses in the telescope in the above problem needed to produce a final image very far from the observer, where vision is most relaxed. Note that a telescope is normally used to view very distant objects.

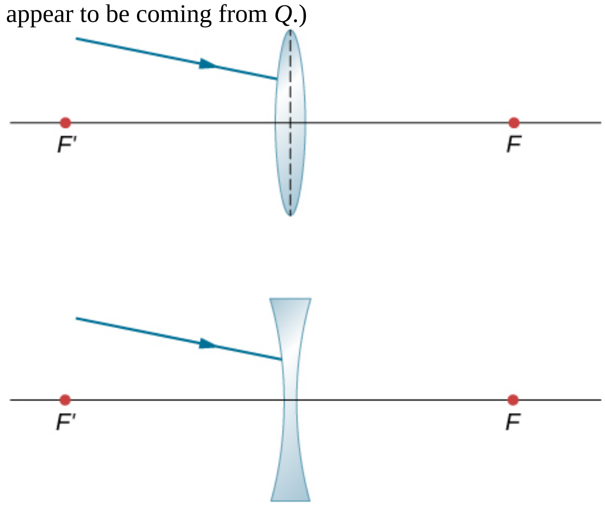
103. A large reflecting telescope has an objective mirror with a 10.0-m radius of curvature. What angular magnification does it produce when a 3.00 m-focal length eyepiece is used?

104. A small telescope has a concave mirror with a 2.00-m radius of curvature for its objective. Its eyepiece is a 4.00 cm-focal length lens. (a) What is the telescope's angular magnification? (b) What angle is subtended by a 25,000 km-diameter sunspot? (c) What is the angle of its telescopic image?

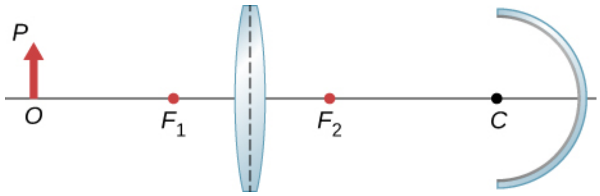
105. A $7.5\times$ binocular produces an angular magnification of -7.50, acting like a telescope. (Mirrors are used to make the image upright.) If the binoculars have objective lenses with a 75.0-cm focal length, what is the focal length of the eyepiece lenses?

106. **Construct Your Own Problem** Consider a telescope of the type used by Galileo, having a convex objective and a concave eyepiece as illustrated in part (a) of **Figure 2.40**. Construct a problem in which you calculate the location and size of the image produced. Among the things to be considered are the focal lengths of the lenses and their relative placements as well as the size and location of the object. Verify that the angular magnification is greater than one. That is, the angle subtended at the eye by the image is greater than the angle subtended by the object.

107. Trace rays to find which way the given ray will emerge after refraction through the thin lens in the following figure. Assume thin-lens approximation. (*Hint*: Pick a point *P* on the given ray in each case. Treat that point as an object. Now, find its image *Q*. Use the rule: All rays on the other side of the lens will either go through *Q* or appear to be coming from *Q*.)



108. Copy and draw rays to find the final image in the following diagram. (*Hint*: Find the intermediate image through lens alone. Use the intermediate image as the object for the mirror and work with the mirror alone to find the final image.)



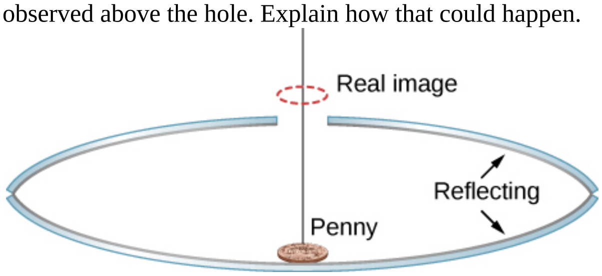
109. A concave mirror of radius of curvature 10 cm is placed 30 cm from a thin convex lens of focal length 15 cm. Find the location and magnification of a small bulb sitting 50 cm from the lens by using the algebraic method.

110. An object of height 3 cm is placed at 25 cm in front of a converging lens of focal length 20 cm. Behind the lens there is a concave mirror of focal length 20 cm. The distance between the lens and the mirror is 5 cm. Find the location, orientation and size of the final image.

111. An object of height 3 cm is placed at a distance of 25 cm in front of a converging lens of focal length 20 cm, to be referred to as the first lens. Behind the lens there is another converging lens of focal length 20 cm placed 10 cm from the first lens. There is a concave mirror of focal length 15 cm placed 50 cm from the second lens. Find the location, orientation, and size of the final image.

112. An object of height 2 cm is placed at 50 cm in front of a diverging lens of focal length 40 cm. Behind the lens, there is a convex mirror of focal length 15 cm placed 30 cm from the converging lens. Find the location, orientation, and size of the final image.

113. Two concave mirrors are placed facing each other. One of them has a small hole in the middle. A penny is placed on the bottom mirror (see the following figure). When you look from the side, a real image of the penny is observed above the hole. Explain how that could happen.



114. A lamp of height 5 cm is placed 40 cm in front of a converging lens of focal length 20 cm. There is a plane mirror 15 cm behind the lens. Where would you find the image when you look in the mirror?

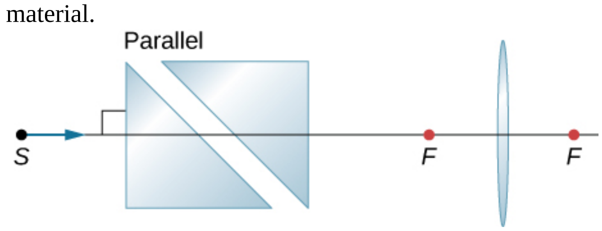
115. Parallel rays from a faraway source strike a converging lens of focal length 20 cm at an angle of 15 degrees with the horizontal direction. Find the vertical position of the real image observed on a screen in the focal plane.

116. Parallel rays from a faraway source strike a diverging lens of focal length 20 cm at an angle of 10 degrees with the horizontal direction. As you look through the lens, where in the vertical plane the image would appear?

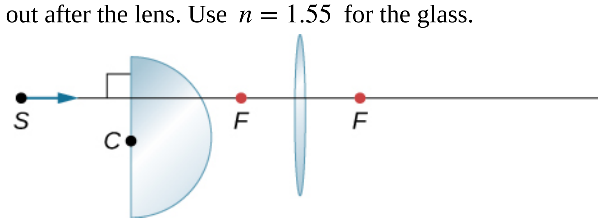
117. A light bulb is placed 10 cm from a plane mirror, which faces a convex mirror of radius of curvature 8 cm. The plane mirror is located at a distance of 30 cm from the vertex of the convex mirror. Find the location of two images in the convex mirror. Are there other images? If so, where are they located?

118. A point source of light is 50 cm in front of a converging lens of focal length 30 cm. A concave mirror with a focal length of 20 cm is placed 25 cm behind the lens. Where does the final image form, and what are its orientation and magnification?

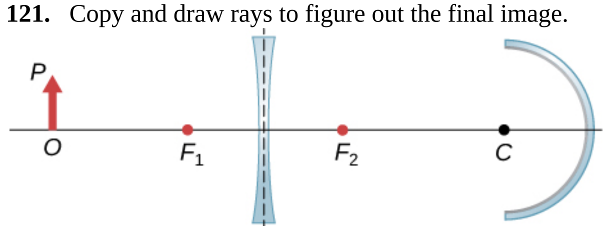
119. Copy and trace to find how a horizontal ray from *S* comes out after the lens. Use $n_{\text{glass}} = 1.5$ for the prism material.



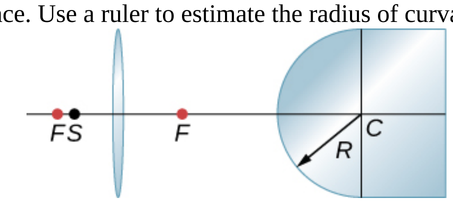
120. Copy and trace how a horizontal ray from *S* comes out after the lens. Use $n = 1.55$ for the glass.



121. Copy and draw rays to figure out the final image.



122. By ray tracing or by calculation, find the place inside the glass where rays from *S* converge as a result of refraction through the lens and the convex air-glass interface. Use a ruler to estimate the radius of curvature.



123. A diverging lens has a focal length of 20 cm. What is the power of the lens in diopters?

124. Two lenses of focal lengths of f_1 and f_2 are glued together with transparent material of negligible thickness. Show that the total power of the two lenses simply add.

125. What will be the angular magnification of a convex lens with the focal length 2.5 cm?

126. What will be the formula for the angular magnification of a convex lens of focal length *f* if the eye is very close to the lens and the near point is located a distance *D* from the eye?

ADDITIONAL PROBLEMS

127. Use a ruler and a protractor to draw rays to find images in the following cases.

(a) A point object located on the axis of a concave mirror located at a point within the focal length from the vertex.
(b) A point object located on the axis of a concave mirror located at a point farther than the focal length from the vertex.
(c) A point object located on the axis of a convex mirror located at a point within the focal length from the vertex.
(d) A point object located on the axis of a convex mirror located at a point farther than the focal length from the vertex.
(e) Repeat (a)–(d) for a point object off the axis.

128. Where should a 3 cm tall object be placed in front of a concave mirror of radius 20 cm so that its image is real and 2 cm tall?

129. A 3 cm tall object is placed 5 cm in front of a convex mirror of radius of curvature 20 cm. Where is the image formed? How tall is the image? What is the orientation of the image?

130. You are looking for a mirror so that you can see a four-fold magnified virtual image of an object when the object is placed 5 cm from the vertex of the mirror. What kind of mirror you will need? What should be the radius of curvature of the mirror?

131. Derive the following equation for a convex mirror:

$$\frac{1}{VO} - \frac{1}{VI} = -\frac{1}{VF},$$

where *VO* is the distance to the object *O* from vertex *V*, *VI* the distance to the image *I* from *V*, and *VF* is the distance to the focal point *F* from *V*. (*Hint*: use two sets of similar triangles.)

132. (a) Draw rays to form the image of a vertical object on the optical axis and farther than the focal point from a converging lens. (b) Use plane geometry in your figure and prove that the magnification *m* is given by $m = \frac{h_\text{i}}{h_\text{o}} = -\frac{d_\text{i}}{d_\text{o}}$.

133. Use another ray-tracing diagram for the same situation as given in the previous problem to derive the thin-lens equation, $\frac{1}{d_\text{o}} + \frac{1}{d_\text{i}} = \frac{1}{f}$.

134. You photograph a 2.0-m-tall person with a camera that has a 5.0 cm-focal length lens. The image on the film must be no more than 2.0 cm high. (a) What is the closest distance the person can stand to the lens? (b) For this distance, what should be the distance from the lens to the film?

135. Find the focal length of a thin plano-convex lens. The front surface of this lens is flat, and the rear surface has a radius of curvature of $R_2 = -35\,\text{cm}$. Assume that the index of refraction of the lens is 1.5.

136. Find the focal length of a meniscus lens with $R_1 = 20\,\text{cm}$ and $R_2 = 15\,\text{cm}$. Assume that the index of refraction of the lens is 1.5.

137. A nearsighted man cannot see objects clearly beyond 20 cm from his eyes. How close must he stand to a mirror in order to see what he is doing when he shaves?

138. A mother sees that her child's contact lens prescription is 0.750 D. What is the child's near point?

139. Repeat the previous problem for glasses that are 2.20 cm from the eyes.

140. The contact-lens prescription for a nearsighted person is −4.00 D and the person has a far point of 22.5 cm. What is the power of the tear layer between the cornea and the lens if the correction is ideal, taking the tear layer into account?

141. **Unreasonable Results** A boy has a near point of 50 cm and a far point of 500 cm. Will a −4.00 D lens correct his far point to infinity?

142. Find the angular magnification of an image by a magnifying glass of $f = 5.0\,\text{cm}$ if the object is placed

$d_o = 4.0\,\text{cm}$ from the lens and the lens is close to the eye.

143. Let objective and eyepiece of a compound microscope have focal lengths of 2.5 cm and 10 cm, respectively and be separated by 12 cm. A $70\text{-}\mu\text{m}$ object is placed 6.0 cm from the objective. How large is the virtual image formed by the objective-eyepiece system?

144. Draw rays to scale to locate the image at the retina if the eye lens has a focal length 2.5 cm and the near point is 24 cm. (*Hint*: Place an object at the near point.)

145. The objective and the eyepiece of a microscope have the focal lengths 3 cm and 10 cm respectively. Decide about the distance between the objective and the eyepiece if we need a $10\times$ magnification from the objective/eyepiece compound system.

146. A far-sighted person has a near point of 100 cm. How far in front or behind the retina does the image of an object placed 25 cm from the eye form? Use the cornea to retina distance of 2.5 cm.

147. A near-sighted person has afar point of 80 cm. (a) What kind of corrective lens the person will need if the lens is to be placed 1.5 cm from the eye? (b) What would be the power of the contact lens needed? Assume distance to contact lens from the eye to be zero.

148. In a reflecting telescope the objective is a concave mirror of radius of curvature 2 m and an eyepiece is a convex lens of focal length 5 cm. Find the apparent size of a 25-m tree at a distance of 10 km that you would perceive when looking through the telescope.

149. Two stars that are $10^9\,\text{km}$ apart are viewed by a telescope and found to be separated by an angle of 10^{-5} radians. If the eyepiece of the telescope has a focal length of 1.5 cm and the objective has a focal length of 3 meters, how far away are the stars from the observer?

150. What is the angular size of the Moon if viewed from a binocular that has a focal length of 1.2 cm for the eyepiece and a focal length of 8 cm for the objective? Use the radius of the moon $1.74 \times 10^6\,\text{m}$ and the distance of the moon from the observer to be $3.8 \times 10^8\,\text{m}$.

151. An unknown planet at a distance of $10^{12}\,\text{m}$ from Earth is observed by a telescope that has a focal length of the eyepiece of 1 cm and a focal length of the objective of 1 m. If the far away planet is seen to subtend an angle of 10^{-5} radian at the eyepiece, what is the size of the planet?